1. Introduction

Offering quantity discounts is a marketing strategy widely used by retailers in several consumer goods markets. The brands in these markets simultaneously offer different-sized packages of the same products. The larger packages of these products, which are otherwise identical, are usually offered at a lower unit price. Two types of quantity discount pricing schedules are used by retailers. In the first, retailers use quantity discounts as a form of price discrimination by permanently offering different package sizes for the same products. In these cases, the pricing schedule is used as a screening mechanism that induces different types of consumers to buy products of different sizes. In this paper, we focus on the second type of schedule: quantity discounts used as a sales promotion instrument. In these cases, in addition to permanently offering a regular-sized package, the retailer also offers a larger package with a quantity discount for a limited period of time. These types of temporary-quantity discounts are used as promotional devices both to stimulate demand and to encourage consumption (“Sun 2005”).

Two price frames for quantity discount promotions are used by retailers and manufacturers in practice. First, the retailer can use multi-unit promotions (for example, “X + 1 free”). In this scenario, the retailer offers a quantity discount if a minimum quantity of a particular stock-keeping unit is purchased. A retailer can conduct this type of promotion without the participation of the manufacturer (“Foubert/Gijsbrechts 2007”). The second option is for the retailer to offer a larger package with a quantity discount (for example, “X for $ Y”), in which the manufacturer supplies the larger packages. Retailer demand for larger package sizes can be stimulated by the manufacturer or may originate from the retailer itself and the decision-making firm can be a manufacturer or a retailer. The manufacturer and the retailer must decide when and for how long to offer the discounts (i.e., they must make timing decisions) and they must also determine the optimal quantity discount. Typically, retailers will address the timing decision together with their manufacturers. The decision regarding the optimal quantity discount, however, must be made by the retailers. Therefore, we assume the perspective of the retailer, who typically sets the prices in both price frames (“Draganska/Klapper/Villas-Boas 2010”). In this paper, we assume that the availability and timing decisions regarding quantity discounts are exogenously influenced and we focus on the optimal size of the quantity discounts.

Retailers use quantity discounts as sales promotion instruments in many product categories. In addition to the regular package size, a larger package size of a brand is offered for only a limited time. This paper addresses the problem of determining the optimal discount (for example, “X + 1 free”) in which the retailer offers a quantity discount if a minimum quantity of a particular stock-keeping unit is purchased. A theoretical model is constructed that takes into account within-brand and cross-brand effects (for example, due to brand-switching), cross-period effects (for example, due to stockpiling) and category-expansion effects (for example, due to increased consumption). From a retailer’s point of view, we determine the optimal quantity discount based on the assumption that the duration of the promotion has been fixed at the beginning of the planning period. We derive optimal policies for retailers and use a comparative static analysis to identify the dependence of quantity discounts upon the key promotion effects of the model. Additionally, we derive meaningful managerial implications for retailers.
Quantity discount promotions likely cause important promotional effects. Because quantity discounts may cause the regular package size to be less popular with buyers, the retailer must evaluate the impact on its profits. Sales of competing products and profits during and after the promotional period in which a retailer offers quantity discounts should be considered.

It has been suggested that the optimal size of a quantity discount depends on many considerations, including within-brand and cross-brand effects (for example, due to brand-switching), cross-period effects (for example, due to stockpiling) and category-expansion effects (for example, due to increased consumption). These promotions have been investigated in the empirical literature primarily in the context of temporary pure-price promotions (for example, “Nijs et al. 2001; Sun/Neslin/Srinivasan 2003; van Heerde/Leeflang/Wittink 2004”), whereas only a few papers have studied the effects on optimal depth in the context of pure-price promotions (for example, “Belot/Jørgensen/Zaccour 2006; Rao 1992; Rao/Thomas 1973; Tellis/Zufryden 1995”).

When offering quantity discount promotions, retailers may find it more challenging to optimize the depth of the quantity discount. Assuming that a retailer offers two brands in a particular category, there are several questions that need to be considered. What will be the optimal quantity discount if the two brands are promoted consecutively? Should the retailer use the same discount for both brands when consumers are stockpiling? Will the results change when the retailer uses quantity discount promotions in a category without brand-switching behaviour? To address these questions, we adapt the promotion effects and construct a theoretical model of demand, which we can use to determine the optimal depth of quantity discounts. Our theoretical model incorporates the main decisions that a retailer faces in determining the size of the quantity discount and enables us to analyse the impact of brand-switching, stockpiling and category-expansion on the profits that a retailer can earn during a predefined period by using quantity discounts. Using these results, we then derive the optimal quantity discount for a scenario in which the retailer promotes two brands and the timing of the quantity discount promotion is fixed at the beginning of the planning period. Comparative statics are used to derive managerial implications and a numerical study is conducted to determine the optimal quantity discount.

To the best of our knowledge, this is the first study to determine the optimal depth of a quantity discount in a temporary promotion. There are many studies in the marketing literature that address issues related to quantity discounts. However, most of those studies addressed the role that quantity discounts play in improving channel coordination in the relationship between manufacturers and their retailers (for example, “Raju/Zhang 2005”). Within an economic model of demand, Allenby et al. (2004) empirically addressed optimal consumer choices given a discrete number of package sizes in a market of strong substitutes. Similarly, Lewis/Singh/Fay (2006) conducted an empirical study that investigated the impact of shipping charges on order incidence and order size. They found that promotions such as free shipping for orders that exceeded a particular size threshold were very effective in generating additional sales. In the two previously mentioned papers, the prices chosen by the retailers were exogenously determined. In contrast, in our theoretical model, the quantity discount pricing strategies are determined endogenously. In a theoretical model, Subramaniam/Gal-Or (2009) evaluated the profitability of offering quantity discounts in a market with competing but differentiated products. They find that the extent of the discounting declined when products were less differentiated. Additionally, discounts decreased if loyalty was greater among heavy users. However, the authors did not consider the dynamic promotion effects that likely occurred.

The remainder of this article is organized as follows. In Section 2, we discuss the possible effects of quantity discounts and summarize the findings. Section 3 presents the model and Section 4 presents a numerical example. In Section 5, we conduct a comparative static analysis to derive the managerial implications of our findings. Section 6 summarizes the main results of the study.

2. The effects of quantity discount promotions

A vast body of research documents the effects of pure-price promotions on consumer purchase behaviour and decomposes sales promotion increases. These promotion effects likely occur when quantity discount promotions are used. Therefore, we appropriate this decomposition framework and use it to analyse quantity discounts, incorporating the promotion effects into our theoretical model. Given the assumption that these promotion effects are estimated, we can then determine the optimal quantity discount. An extensive body of empirical research has established that a price promotion should lead to an increase in own-brand sales of the promoted brand during the promotional period (for example, “Blattberg/Neslin 1990; Gupta 1988”). The standard decomposition splits the increase in own-brand sales into three parts: cross-brand effects (for example, due to brand-switching), cross-period effects (for example, due to stockpiling) and category-expansion effects (for example, due to increased consumption). In the following section, we summarize the main findings regarding the decomposition of sales promotion increases and discuss...
the potential effects within a quantity discount promotion setting.

Cross-brand and within-brand sales effect

Typically, a promotion causes a decrease in current sales for competing brands within the category. Several researchers have decomposed sales promotion elasticities based on household scanner-panel data (“Bell/Chiang/Padmanabhan 1999; Bucklin/Gupta/Siddarth 1998; Chiang 1991; Chintagunta 1993; Gupta 1988”). Among these decomposition studies, we found that, on average, brand-switching effects account for the vast majority of the total elasticity. On average, approximately 74 % of the total elasticity was attributed to this effect. In a dynamic structural model, Sun/Neslin/Srinivasan (2003) measured the impact of promotions on brand-switching when consumers were forward looking. The results indicated that 56 % of the short-term effects of sales could be attributed to brand-switching effects. In a previous study, van Heerde/Gupta/Wittink (2003) derived an analytical expression that transformed this elasticity decomposition into a decomposition of unit sales effects. They found that approximately 33 % of the sales promotion increases were due to brand-switching effects. By using aggregate weekly scanner data, van Heerde/Leeflang/Wittink (2004) showed that the brand-switching effect created 35 % of the own-brand sales effect. Only a few studies have documented brand-switching effects associated with quantity discounts. In a random-utility framework for consumer choice under quantity discounts, Allenby et al. (2004) demonstrated that price reduction within a brand caused significant within-brand effects (i.e., consumers may buy another product by the same brand) but limited cross-brand effects (i.e., consumers may not buy a competitor’s product). Foubert/Gijsbrechts (2007) found that promotional bundling was particularly effective in stimulating brand-switching behaviour. Thus, we expect that quantity discount promotions will induce brand-switching but that the within-brand effect will be larger than the cross-brand effect.

Cross-period sales effect

Household inventory models based on household scanner-panel data have predicted that in the weeks after a promotion, consumers will purchase less if every other input remains constant. This phenomenon, in turn, results in a decline in demand in the weeks following a promotion (for example, “Bell/Chiang/Padmanabhan 1999; Blattberg/Eppen/Lieberman 1981; Chiang 1991; Currim/Schneider 1991; Gupta 1991; Gupta 1988; Neslin/Henderson/Quelch 1985”). Using aggregate weekly scanner data, van Heerde/Leeflang/Wittink (2004) determined that the estimated cross-period effect of sales promotion increases was 44 %. These results were consistent with those of household-level studies. Hendel/Nevo (2003) showed that the cross-period effect was present in aggregate data once they controlled for additional promotional activities such as features and displays. Macé/Neslin (2004) found that 22 % of the sales promotion increase was attributable to cross-period effects. To the best of our knowledge, there are no papers that have studied cross-period effects when retailers engaged in quantity discount promotions. We expect that consumers will purchase less in the weeks after a quantity discount promotion as a result of stockpiling.

Category-expansion effect

Category-expansion is mainly driven by increased consumption. The first empirical paper to study the promotion effect on consumption is Ailawadi/Neslin (1998). The authors established that 35 % of the short-term sales increase could be represented by an increase in consumption. Using a dynamic structural model with endogenous consumption, Sun (2005) showed that consumption increased during price promotions and that 33 % of the sales increase could be attributed to an increase in consumption. The decomposition framework presented by van Heerde/Gupta/Wittink (2003) estimated a category-expansion effect of 20 %. Within a times-series framework, Pauwels/Hanssens/Siddarth (2002) also estimated that price promotions demonstrated a category-expansion effect (of approximately 62 %). Furthermore, these results were consistent with the empirical work of Nijs et al. (2001) and Dekimpe/Hanssens/Silva-Risso (1999). Foubert/Gijsbrechts (2007) estimated that a bundle promotion’s potential to cause consumers to buy more in a particular category was limited. Therefore, we assume the existence of category-expansion effects as a result of quantity discount promotions, but we also assume that their extent is limited.

These promotion effects are well documented for pure-price promotions and it appear that these effects are likely to occur when retailers use quantity discount promotions. We decompose the own-brand sales effect (the effect on the sale of large packages) into four parts: within-brand effects (due to within-brand-switching), cross-brand effects (due to cross-brand-switching), cross-period effects (due to stockpiling) and category-expansion effects (due to increased consumption). We acknowledge these effects and derive the optimal quantity discount in the next section.

3. Model of quantity discounts

The model we use to optimise the quantity discount decision parameter (d) will be discussed in this section. We follow the model construction process for pure-price promotions employed by Belot/Jørgensen/Zaccour (2006) and use the model in the quantity discount promotion setting. We determine the optimal quantity discount on the assumption that the previously discussed promotion effects were given. The retailer is assumed to be a monopolist, which rules out inter-store competition and we assume that the retailer offers two brands (i = 1, 2) in a regular-sized package in a particular product category. The
3.1. Demand and profit functions

This section describes the demand and profit conditions in the five time intervals. We use linear demand functions for two reasons. First, linear demand functions are mathematically tractable. Second, studies in the analytical literature have shown that the results of optimization problems depend on the slope of the demand function instead of whether the demand functions are linear (for example, “Lee/Staelin 1997; Purohit/Staelin 1994”).

The retailer is concerned with profit maximization. The symbol \( p_i^l \) denotes the regular prices of the regular-sized packages of Brand \( i \), and \( w_i^l \) denotes the wholesale price. Therefore, \( m_i^l = (p_i^l - w_i^l) \) is the margin of Brand \( i \). The price of the large-sized package of Brand \( i \) without a quantity discount is denoted by \( p_i^l \). “\( S \)” is the index of a regular package size, “\( L \)” the index of the large package size. The demand rates in the five time intervals are denoted by \( q_i^l \) and \( q_i^L \).

First time interval \([0, \theta_1]\):
The demand rates for the regular-sized packages by the two brands during the time interval \([0, \theta_1]\) are unaffected by the promotion effects and are given by

\[
q_i^l > 0; \quad q_i^L > 0. \tag{1}
\]

Multiplying the demand rates by the margins for the regular-sized packages by both brands yields the profit

\[
K = (p_1^l - w_1^l)q_1^1 + (p_2^l - w_2^l)q_2^1, \tag{2}
\]

which can be viewed as the retailer’s baseline profit. The profit rate during the time interval \([0, \theta_1]\) equals \( \theta_1 K \).

Second time interval \([\theta_1, \theta_2]\):
During the promotion period for Brand 1, the demand rates for the regular packages by Brand 1 and 2 and the demand rates for the large packages by Brand 1 are affected by the depth of the quantity discount for Brand 1 \( (d_1) \). The symbol \( \epsilon \) denotes the percentage increase in the package size. The price of the large package by Brand 1 without a quantity discount is given by \( p_1^l = (1 + \epsilon)p_i^l \).

The quantity discount for Brand 1 is defined as a discount from the regular price (cents off the regular price). Therefore, the promotion price for the large package by Brand 1 \( p_1^l - d_1 \). The quantity discount acts as the retailer’s decision variable. The demand rates during the promotion period for the large package by Brand 1, the regular package by Brand 1 and the regular package by Brand 2 are given by the following linear demand functions

\[
\begin{align*}
\tilde{q}_i^l (d_1) &= \beta_i d_1, \\
\tilde{q}_i^L (d_1) &= \tilde{q}_i^l - \gamma_i d_1, \\
\tilde{q}_i^L (d_1) &= \tilde{q}_i^L - \epsilon_2 d_1 \quad \text{where} \quad \beta_i \geq 0, \gamma_i \geq 0, \epsilon_2 \geq 0, \beta_i = \gamma_i + \epsilon_2 + \sigma_{iw} + \lambda_1. \tag{3}
\end{align*}
\]

In (3), \( \beta_i \) is the own-brand sales effect that indicates the marginal impact of the quantity discount on the demand for the large package by Brand 1. A higher quantity discount on Brand 1 results in higher demand during the
promotion period. The term $\gamma_1$ represents the within-brand effect: the marginal impact of the quantity discount on demand for the regular package by the same brand. A higher quantity discount results in lower demand during the promotion period. The parameter $\varepsilon_3$ reflects the cross-brand effect: the decrease in competitive brand prices during the time interval $(T, T+1)$. The demand rate in (3) clearly shows that the retailer faces a tradeoff between the increased demand for the large package and the decreased demand for the regular package when using quantity discounts as a promotional instrument. Multiplying the demand rates by the corresponding margins provides the profit

$$K(d_\theta) = (p_i^1 - w_i^1)[q_1^\theta - \gamma_1 d_\theta] + (p_i^2 - w_i^2)[q_2^\theta - \gamma_2 d_\theta].$$

The wholesale price of the large package by Brand 1 is given by $w_i^1 = (1 + x)w_i^2$. The retailer’s profit in the time interval $(\theta_2, \theta_3)$ equals $(\theta_2 - \theta_3)K$.

Third time interval $(\theta_2, \eta_1)$: In the post-promotion period for Brand 1, we account for cross-period effects. Within the promotion period, consumers buy more to meet their current consumption needs and to stockpile for future consumption. We assume that post-promotion demand for Brands 1 and 2 are affected by the depth of the Brand 1 quantity discount so that a steeper quantity discount results in more stockpiling and lower post-promotion demand for Brands 1 and 2. The demand rates for the regular packages by both brands during the time interval $(\theta_2, \eta_1)$ are given by

$$\hat{q}_1^\theta(d_\theta) = \bar{q}_1^\theta - \sigma_{1d}d_\theta,
\hat{q}_2^\theta(d_\theta) = \bar{q}_2^\theta - \sigma_{2d}d_\theta$$

where $\sigma_{1d} > \sigma_{2d} > 0$, in which $\sigma_{1d}$ is the cross-period effect on Brand 1 and $\sigma_{2d}$ is the cross-period effect on Brand 2. We subtract the cross-period effect from the regular demand for the small packages by Brands 1 and 2, which results in a decrease in their demand rates. We expect that $\sigma_{1d} > \sigma_{2d}$, that is, the quantity discount promotion by Brand 1 has a stronger effect on its own demand than on that of Brand 2. The post-promotion profit becomes

$$K(d_\theta) = (p_i^1 - w_i^1)[\bar{q}_1^\theta - \sigma_{1d}d_\theta] + (p_i^2 - w_i^2)[\bar{q}_2^\theta - \sigma_{2d}d_\theta].$$

and the profit in the time interval $(\theta_2, \eta_1)$ equals $(\eta_1 - \eta_2)K$. Note that $K \leq K$, which means that the post-promotion profit will not exceed the baseline profit.

Fourth time interval $(\eta_1, \eta_2)$: During the promotion period for Brand 2, the demand rates for the regular packages by Brands 1 and 2 and those for the large package by Brand 2 are affected by the depth of the quantity discount offered by Brand 2 $(d_\eta)$. The equation $p_i^2 = (1 + x)p_i^1$ denotes the price of the large package by Brand 2 without a quantity discount, and $p_i^2 - d_\eta$ denotes the promotion price of the large package by Brand 2. We assume the same percentage increase in the package size as for Brand 1. The demand rates during the time interval $(\eta_1, \eta_2)$ are given by

$$\hat{q}_1^\eta(d_\eta) = \bar{q}_1^\eta - \varepsilon_1 d_\eta - \sigma_{1d}d_\eta,$n\hat{q}_2^\eta(d_\eta) = \bar{q}_2^\eta - \varepsilon_2 d_\eta$$

where $\beta_2 \geq 0, \gamma_2 \geq 0, \varepsilon_1, \beta_2 \geq 0$. In (7), $\beta_2$ measures the marginal impact of the quantity discount on demand for the large package by Brand 2 (own-brand sales effect). The term $\gamma_2$ is the within-brand effect and the parameter $\varepsilon_2$ reflects the cross-brand effect. Note that the cross-period effects of the quantity discount promotion offered by Brand 1 are also present in the time interval $(\eta_1, \eta_2)$ through the term $\sigma_{1d}d_\eta$. For simplicity, we don’t assume this type of effect for Brand 2, although the model could easily be extended. Thus, the profit is

$$K(d_\eta) = (p_i^1 - w_i^1)[\bar{q}_1^\eta - \varepsilon_1 d_\eta - \sigma_{1d}d_\eta] + (p_i^2 - w_i^2)[\bar{q}_2^\eta - \gamma_2 d_\eta],$$

The profit in the time interval $(\eta_1, \eta_2)$ equals $(\eta_2 - \eta_1)K$.

Fifth time interval $(\eta_2, T)$: In the last period, consumers can only buy the regular packages by both brands. In the post-promotion period for Brand 2, we account for cross-period effects. We assume that post-promotion demand for Brands 1 and 2 are affected by the depth of the quantity discount offered by Brand 2. The demand rates for the regular packages by both brands during the time interval $(\eta_2, T)$ are given by

$$\hat{q}_1^\eta(d_\eta) = \bar{q}_1^\eta - \sigma_{1d}d_\eta,$n\hat{q}_2^\eta(d_\eta) = \bar{q}_2^\eta - \sigma_{2d}d_\eta$$

where $\sigma_{2d} > \sigma_{1d} > 0$, in which $\sigma_{2d}$ is the cross-period effect on Brand 2 and $\sigma_{1d}$ is the cross-period effect on Brand 1. We subtract the cross-period effect from the regular demand for the small packages by Brands 1 and 2, creating a decrease in their demand rates. We predict that $\sigma_{2d} > \sigma_{1d}$. The post-promotion profit becomes

$$K(d_\eta) = (p_i^1 - w_i^1)[\bar{q}_1^\eta - \sigma_{1d}d_\eta] + (p_i^2 - w_i^2)[\bar{q}_2^\eta - \sigma_{2d}d_\eta].$$

and the profit in the time interval $(\eta_2, T)$ equals $(T - \eta_2)K$.

3.2. Optimal quantity discounts

The retailer’s face a multi-stage decision problem when implementing optimal quantity discounts. In the second period, the retailer offers a quantity discount on the Brand 1 product $(d_\eta)$; in the fourth period, the retailer offers the discount on the Brand 2 product $(d_\eta)$. Note that we determine the optimal quantity discount based on the assumption that the duration of the promotions $(\theta_2 - \theta_1)$ and $(\eta_2 - \eta_1)$ is fixed at time zero. The problem can be solved by means of backward induction starting at time $t = \eta_2$ (see appendix). Promotions are typically short-term activities and we can safely omit discounting future profits; total profits are realized within a few days or weeks. The profit function for the retailer during the planning period $(0, T)$ is given by
\[ \Pi(\theta_1, \theta_2, \eta_1, \eta_2, d_\theta, d_\eta) = \theta_1 K + (\theta_2 - \theta_1)K + (\eta_2 - \eta_1)K + (\eta_2 - \eta_1)K. \]  

Inserting the profits from (2), (4), (6), (8) and (10) into (11), one derives a profit function of

\[ \Pi(\theta_1, \theta_2, \eta_1, \eta_2, d_\theta, d_\eta) = \theta_1 [m_1^2 \gamma_1^2 + (m_2^2 \gamma_2^2] + (m_1^2 - d_\theta)\sigma_1 d_\theta + m_1^2 \sigma_1 d_\theta \]  
\[ + (m_2^2 - \theta_2)\sigma_2 d_\theta + m_2^2 \sigma_2 d_\theta + (T - \eta_2)(\eta_1 \sigma_1 d_\theta + m_2^2 \sigma_2 d_\theta)]. \]  

The optimal quantity discount for Brand 1 is given by

\[ d_\theta = \frac{1}{2\beta_1}(\theta_2 - \theta_1) \times [m_1^2 \gamma_1^2(\gamma_1 - \gamma_2) + \sigma_1 d_\theta(\eta_1 - \eta_2)] - m_1^2 \sigma_1 d_\theta \]  
and the optimal quantity discount for Brand 2 is given by

\[ d_\eta = \frac{1}{2\beta_2}(\eta_1 - \eta_2) \times [m_2^2 \gamma_2^2(\gamma_1 - \gamma_2) + \sigma_2 d_\eta(T - \eta_2)] - m_2^2 \sigma_2 d_\eta. \]

Equations (13) and (14) show that the optimal quantity discount is influenced by promotion effects, prices and time instants. An important property of our results is that they are independent of the absolute value of the promotion effects. Therefore, the optimal quantity discount only depends on the percentage decomposition of the own-brand sales effect. We interpret the results using a numerical example and comparative statics in the sections that follow.

4. Numerical example

In this section, we illustrate the optimal quantity discounts for the two brands indicated by the analytical results in (13) and (14) using a numerical example. For illustration purposes, we use the prices, marginal costs and market shares of the soft drink industry. With fixed time instants and own-brand, within-brand, cross-brand and cross-period effects, the optimal quantity discounts are calculated and the corresponding demand rates and profits are shown.

A retailer offers two brands in the soft drink product category (Brand 1: Coca-Cola; Brand 2: Pepsi) and only one brand is promoted at a particular time by offering an additional larger package size and a quantity discount. The retailer is interested in determining the optimal quantity discounts for both brands to maximize profits. The parameters of our numerical example are displayed in Tab. 1. Within the retailer’s promotion planning period of 30 days \( T \), the retailer offers the two brands’ products in a regular package size (for example, 1 litre) at a regular price of \$ .72 for Brand 1 and \$ .70 for Brand 2. The wholesale prices are \$ .39 for Brand 1 and \$ .33 for Brand 2. Arbitrarily, the market size is assumed to be 1,000. Given the market share of both brands (Brand 1: 38 % and Brand 2: 32 %), the baseline demand rate is 380 units for Brand 1 and 320 units for Brand 2. Therefore, the retailer’s baseline profit is \$ 243.80 \( (= 380 \times (0.72 - 0.39) + 320 \times (0.70 - 0.33)) \). The promotion period for Brand 1 starts on Day 10 and ends on Day 15. During this period, the retailer offers a double-sized item by Brand 1 \( (x = 1) \) with a quantity discount \( (d_\theta) \). The price of the large-sized package (for example, a 2-liter package) without a quantity discount amounts to \$ 1.44. The post-promotion period lasts 5 days and ends on Day 20. The promotion period for Brand 2 starts on Day 20 and ends on Day 25. During this period, the retailer offers a double-sized item by Brand 2 \( (x = 1) \) with a quantity discount \( (d_\eta) \). The price of the large package without a quantity discount is \$ 1.40. On the basis of our literature review in Section 2, we assume the following decomposition of the own-brand effect \( (p_1 = 400: 100 \%) \), the within-brand effect \( (p_2 = 100: 25 \%) \), cross-brand effect \( (p_3 = 100: 25 \%) \), the cross-period effect on the company’s own demand \( (p_4 = 50: 15 \%) \), the cross-period effect on competitor demand \( (p_5 = 50: 10\%) \) and the category-expansion effect \( (p_6 = 120: 30 \%) \).

The main results of the numerical example are as follows. Using the analytical expression in (13) and (14), we obtain an optimal quantity discount for Brand 1 of \( d_\theta = .18 \) and \( d_\eta = .24 \) for Brand 2. Thus, the optimal prices of the large packages by Brands 1 and 2 are \( p_1^* - d_\theta = 1.26 \) and \( p_2^* - d_\eta = 1.16 \).

The demand rates in the four periods are shown in Tab. 2. In the first time interval, the demand rates for the two regular-sized packages are 380 and 320 units, respectively. Therefore, the total baseline demand rate for the retailer is 700. Within the second time interval, the demand rates for the regular-sized items decreases to 361.63 (Brand 1) and 301.33 (Brand 2). The decrease in demand is caused by consumers switching from regular-sized items to the promoted item (within-brand and cross-brand effects). At the same time, the demand for the large package is 73.50 (Brand 1). Within this time interval, the total demand is 36.75 units higher than in the first time interval. During the third time interval, the demand rate for the regular-sized package by Brand 1 decreases to 368.98 units in comparison to the first time interval. The demand rate for the regular package by Brand 2 decreases to 316.33 units. This decrease in demand is caused by consumer stockpiling. In the fourth time interval, the demand rate for the regular-sized items decreases to 344.33 (Brand 1) and 295.35 (Brand 2) in comparison to the first time interval. The demand for the large package is 98.60 (Brand 2). In the fifth time interval, the regular-sized packages exhibit demand below the baseline because of cross-period effects. The total profit during the planning period is \$ 7,768.99. If the retailer does not use quantity discounts as a promotional instrument, the profit will be \$ 7,557.80.
Based on the parameters of our numerical example, the impact of the changes in the promotion effects, prices and time instants on the optimal quantity discount will now be evaluated in more detail. In assessing the impact of promotion effects on optimal quantity discounts, we vary the values of $\beta_i$ (own-brand effect), $\gamma_i$ (within-brand effect), $\varepsilon_i$ (cross-brand effect) and $\sigma_{1i}, \sigma_{2i}$ (cross-period effects) and hold the other parameters constant. Fig. 1 depicts the results of this sensitivity analysis with respect to the promotion effects. The solid line is the optimal quantity discount for Brand 1 and the dashed line is the optimal quantity discount for Brand 2. In both cases, the optimal quantity discount is an increasing function of the own-brand effect. We vary the value of the own-brand effect between 280 and 1,000. Note that if the own-brand effect for both brands is 280, the category-expansion effect will be zero; i.e., the demand rate for the introduced product will be solely generated by consu-
mbers who switch from the regular-sized items to the promoted item. If the own-brand effect increases (approximately 280) with a subsequent increase in category expansion, the optimal quantity discount will also increase but will do so at a decreasing rate. The value of the within-brand and cross-brand effects varies between 0 and 200; thus, as these effects rise, the optimal quantity discount will decrease in a linear fashion. If, for example, the value of the within-brand effect is 0, the optimal quantity discount for Brand 1 will be .22 and will be .29 for Brand 2. The optimal quantity discount is also a negative linear function of the cross-period effect on the demand for the brand. As the stockpiling effects increase, the optimal quantity discount will decrease linearly.

The sensitivity of optimal quantity discounts to product prices is shown in Fig. 2. As in the first example, we vary the values of the prices and hold the other parameters constant. The optimal quantity discounts for Brands 1 and 2 are a negative linear function of the prices of the regular packages by Brand 1 and Brand 2. The optimal quantity discount function for Brand 1, which is subject to the price of the regular-sized item of the same brand, is steeper. In particular, the quantity discount should decrease with the increase in the price of the regular package. This concept is derived from our theoretical model, in which increasing the price of a product also increases its margin. If the margin is high, the retailer should only provide a low level of incentive to switch to the promoted item. The right panel of Fig. 2 shows the effect of competitor prices on the optimal quantity discount. Again, the function is downward sloping.

The impact of time instants on the optimal quantity discount for Brand 1 is illustrated in Fig. 3 and the same in-
formation for Brand 2 is presented in Fig. 4. We vary the value of the start of the quantity discount promotion by Brand 1 (θ₁) between 0 and 14. Note that the initial value of the parameter θ₁ is 10 (dashed line). If we increase the length of the promotion period (θ₁ smaller than 10), the optimal quantity discount will increase slightly. The effect will be larger when the promotion period decreases (θ₁ larger than 10). In this case, the optimal quantity discount will decrease by a large degree. The same effect emerges for the optimal quantity discount subject to the end of the promotion period (θ₂). Note that the initial value of the parameter θ₂ is 15 (dashed line). If the length of the promotion period increases, the length of the post-promotion period simultaneously decreases (θ₂ larger than 15) and the optimal quantity discount will increase. In contrast, the optimal quantity discount should decrease when the length of the post-promotion period increases (θ₂ larger than 20). A similar pattern can be observed for the optimal quantity discount for Brand 2 in Fig. 4.

5. Comparative statics

This section describes the comparative static analysis of optimal quantity discounts in different market scenarios involving alternative effects of time instants, prices and promotion effects. From a policy viewpoint, it is important to determine how the optimal quantity discount should change with market conditions. The comparative statics approach is particularly useful in these cases because the store manager can determine how optimal marketing policy responds before market conditions shift. We conduct the comparative static analyses using the first-order conditions for optimal quantity discounts in (13) and (14). We focus on solutions for which the optimal quantity discounts and the corresponding demand rates are positive. The following six propositions regarding the covariates of optimal quantity discounts emerged from the preceding analysis (proofs are given in the appendix):

**Proposition 1:** As the length of the promotion increases, the optimal quantity discount increases.

In our model, the retailer fixes the time instants at the beginning of the planning period. We attempt to determine the effect of the length of a quantity discount promotion on the retailer’s optimal quantity discount policy. For example, the retailer may want to introduce a larger package for a particular brand item for two weeks instead of one week. Should the retailer offer a higher or lower quantity discount? Quantity discount promotion products are often advertised using slogans such as “only for a short time” and last just one or two weeks to prevent the consumers from adjusting to the new price levels. The comparative static analysis shows that if the quantity discount promotion lasts longer, the retailer should offer a higher quantity discount. Increasing the length of the quantity discount promotion lessens the stockpiling effects. When a product is always offered at a discount, there is no need for consumers to stockpile.
Proposition 2: As the length of the post-promotion period increases, the optimal quantity discount decreases.

If the post-promotion period is longer, the retailer should offer a lower quantity discount. The reasoning governing this concept is similar to Proposition 1: a longer post-promotion period increases consumer stockpiling behaviour. Consumers purchase more items during the promotion at a lower unit price, planning to consume them later.

Proposition 3: As the price of the regular-sized item by Brand 1 (Brand 2) increases, the optimal quantity discount decreases.

Increasing the price of the regular package by Brand 1 or Brand 2 decreases the optimal quantity discount for Brand 1 or 2. Therefore, the quantity discounts for Brands 1 and 2 should be smaller when the price of the regular package is higher. Increasing retail prices increases the margin. Therefore, a high-margin brand should only use small quantity discounts to give a small incentive to consumers to switch to the promoted brand. If, for example, Brand 1 introduces a large package with a quantity discount, some buyers who normally purchase the regular-sized packages by Brand 1 and Brand 2 will switch to the new, larger product by Brand 1. They will pay the discounted price, whereas if they had instead purchased the small package by Brand 1, they would have paid the regular price with a higher margin.

Proposition 4: As the price of the regular-sized item by Brand 1 (Brand 2) increases, the optimal quantity discount for Brand 2 (Brand 1) decreases.

Increasing the price of the regular package by Brand 1 (Brand 2) decreases the optimal quantity discount for Brand 2 (Brand 1) as a result of brand substitution. Consumers have a low incentive to switch from high-margin brands to the discounted brand.

Proposition 5: As brand-switching effects (within-brand and cross-brand effects) increase, the optimal quantity discount decreases.

During a promotion, some consumers will switch from a non-promoted brand to the promoted brand or item. The quantity discount on a large package should be smaller when the promotion will negatively affect the current demand level for the regular-sized package by the same brand and that of the competitor. Thus, if within-brand and cross-brand effects are significant, the retailer should offer moderate discounts or even refrain from offering promotions.

Proposition 6: As the cross-period effect increases, the optimal quantity discount decreases.

When retailers offer a quantity discount, consumers may tend to stockpile if they are not opposed to storing the products. This behaviour generates an interesting question: what is the effect of the consumers' tendency to stockpile on a retailer's optimal quantity discount policy? The comparative static analysis shows that if cross-period effects are significant, the retailer should offer a lower quantity discount because as the cross-period effect increases, consumers buy more during the promotion period but use what they buy to create a stockpile for future consumption. This behaviour results in a decrease in post-promotion demand for regular-sized packages. Therefore, a greater cross-period effect results in a greater decrease in demand.

6. Conclusion

Marketing managers frequently create pricing policies involving substantial quantity discounts. We considered a case in which retailers used quantity discounts as a sales promotion instrument. The retailer introduced large packages of items by a particular brand for a short period with a quantity discount. This paper addresses the problem of determining the optimal depth of the quantity discounts for brands in a product category. We present a theoretical model that takes into account own-brand, within-brand, cross-brand, cross-period and category-expansion effects.

Comparative static analysis is essential when studying the dependence of optimal quantity discounts on promotion effects, prices and time instants. The essential results can be summarised as follows:

- If the length of the quantity discount promotion is increased, the retailer should offer a higher quantity discount.
- If the length of the post-promotion period is decreased, the retailer should offer a lower quantity discount.
- Increasing the price of the regular package by Brand 1 or 2 decreases the optimal quantity discount for Brand 1 or 2.
- The quantity discount should be smaller if a promotion by this brand will negatively affect the current demand for the regular-sized packages by the same brand and competing brands.
- If cross-period effects are significant, the retailer should offer a lower quantity discount.

Finally, our research has several limitations that offer some opportunities for additional research. First, we deliberately simplified our theoretical approach to allow for more analytical insights. Some of our assumptions can be adjusted to address more complex and realistic situations if we consider such factors as (a) nonlinear demand functions, (b) cases in which there are more than two brands in a category or (c) cases in which there is more than one quantity discount promotion for each brand. Second, further research could determine the strategic implications of quantity discount promotions in the manufacturer-retailer channel. For example, it would be interesting to analyse the optimal responses of manufactur-
ers to retailer-controlled promotions. Third, empirical work could estimate the effects of quantity discount promotions on demand (cross-brand, cross-period and category-expansion effects). All of the previously mentioned complications would make the analytical model more realistic but also less tractable by potentially limiting the ability of researchers to derive additional insights from their analyses.

Appendix A

The solution of the dynamic programming problem is presented below. \Pi_i^t denotes the value function at stage \(i \in \{1,2,3,4\} \). The value function measures the optimal profit at stage \(i \) and will be calculated in backward time:

**Stage 5**: Starts at time \(t_1^i\), Determine \(\Pi_i^t\)

\[
\Pi_i^t = (T - t_1^i)K = (T - t_1^i)[(m_i^t[\sigma_{t_1^i} - \sigma_{t_1^i}d_{t_1^i}])
\]

\[
+ (m_i^{t - 1}[\sigma_{t_1^{t - 1}} - \sigma_{t_1^{t - 1}}d_{t_1^{t - 1}}])]
\]

(4.1)

**Stage 4**: Starts at time \(t_1\), Determine \(\Pi_i^t\)

\[
\Pi_i^t = \max_{x_{t_1}} \{\Pi_i^{t - 1} - \gamma_1(t_{t_1^i} - t_{t_1^i - 1})\}
\]

\[
+ \{m_i^{t - 1}[\sigma_{t_1^{t - 1}} - \sigma_{t_1^{t - 1}}d_{t_1^{t - 1}}] + \Pi_i^{t - 1}\}
\]

(A.2)

Performing the maximization on the right-hand side then yields the optimal discount of brand 2:

\[
d_2 = \frac{1}{2\beta_2(t_1 - t_1)} \times \{m_i[\gamma_1(t_1 - t_1) + \sigma_{t_1} (T - t_1)] + m_i^t[\epsilon_1(t_1 - t_1)
\]

\[
+ \sigma_{t_1} (T - t_1)] + m_i^t[\beta_1(t_1 - t_1)]\}
\]

(4.3)

**Stage 3**: Starts at time \(t_1\), Determine \(\Pi_i^t\)

\[
\Pi_i^t = (\eta_{t_1} - \theta_{t_1})K + \Pi_i^t d_2^t = (\eta_{t_1} - \theta_{t_1})[\Pi_i^{t - 1} + \Pi_i^t d_2^t]
\]

\[
+ (m_i^{t - 1}[\sigma_{t_1^{t - 1}} - \sigma_{t_1^{t - 1}}d_{t_1^{t - 1}}])]
\]

(4.4)

**Stage 2**: Starts at time \(t_1\), Determine \(\Pi_i^t\)

\[
\Pi_i^t = \max_{\theta_{t_1}} \{\Pi_i^{t - 1} - \gamma_1(t_{t_1^i} - t_{t_1^i - 1})\}
\]

\[
+ \{m_i^{t - 1}[\sigma_{t_1^{t - 1}} - \sigma_{t_1^{t - 1}}d_{t_1^{t - 1}}] + \Pi_i^{t - 1}\}
\]

(A.5)

Performing the maximization on the right-hand side then yields the optimal discount of brand 1:

\[
d_2 = \frac{1}{2\beta_1(t_1 - t_1)} \times \{m_i[\gamma_1(t_1 - t_1) + \sigma_{t_1} (T - t_1)] + m_i^t[\epsilon_1(t_1 - t_1)
\]

\[
+ \sigma_{t_1} (T - t_1)] + m_i^t[\beta_1(t_1 - t_1)]\}
\]

(4.6)

**Stage 1**: Starts at time 0. Determine \(\Pi_i^t\)

\[
\Pi_i^t = \theta_{t_1}K + \Pi_i^t (d_2^t d_2^t)
\]

\[
\theta_{t_1}(m_i^t[\sigma_{t_1^i} + m_i^{t - 1}[\sigma_{t_1^{t - 1}}])
\]

(4.7)

Appendix B

**Proposition 1**: As the length of the promotion increases, the optimal quantity discount increases.

\[
\frac{\partial d_2^t}{\partial \theta_{t_1}} = \frac{1}{2\beta_2(t_1 - t_1)} \{m_i^t[\sigma_{t_1} (T - t_1)] + m_i^{t - 1}[\sigma_{t_1}(T - t_1)]\} > 0
\]

(B.1)

\[
\frac{\partial d_2^t}{\partial \theta_{t_1}} = \frac{1}{2\beta_2(t_1 - t_1)} \{m_i^t[\sigma_{t_1} + m_i^{t - 1}[\sigma_{t_1}]) (T - t_1)\} > 0
\]

(B.2)

**Proposition 2**: As the length of the post-promotion period increases, the optimal quantity discount decreases.

References


Keywords
Nonlinear Pricing; Optimal Quantity Discount; Retailing; Promotion.