Statistical and Managerial Relevance of Aggregation Level and Heterogeneity in Sales Response Models

by Harald Hruschka

Sales response models studied here differ both with regard to aggregation level (chain- or store-level) and consideration of heterogeneous effects of prices. In contrast to related previous publications the present paper evaluates models not only with respect to statistical criteria like model fit and bias of coefficients, but also with respect to their ability to improve decision-making in terms of expected profits. For chain-level models aggregate error and uncertainty about coefficients and price elasticities are much higher. Chain-level elasticities differ from those inferred on the basis of store-level models for three out of nine brands. For three out of nine brands, expected profits are higher if pricing is based on analyzing store-level data. Profit differences across model types can be traced back to differences of a nonlinear function of elasticities. No profit differences are found between homogeneous and heterogeneous models. Because of lower costs compared to its heterogeneous counterpart, the homogeneous store-level model turns out to be a preferable alternative from a managerial point of view.

1. Introduction

Common sense seems to suggest that sales response models should be specified at the same level of aggregation at which decisions are made (Hanssens/Parsons/Schultz 2001; Leebleng et al. 2000). For example, key account managers in manufacturing firms selling packaged consumer goods who behave accordingly see no need to use store-specific data. Instead they base pricing on results which are obtained by analyzing data on aggregate sales and average prices for any given retail chain. Company policies of retail firms often oblige their category managers to set uniform prices, i.e. prices that do not differ between stores. Being interested in effects that different prices have on total sales and total profits, category managers might think that it is sufficient to look at effects of average prices on total sales and ignore store-specific effects.

The fact that aggregate sales response models often suffer from aggregation bias runs counter to this common sense reasoning. Aggregation bias means that parameter estimates obtained from an aggregate model differ from those of a store-level model to a noticeable extent. Leeflang et al. (2000) assert that store-level data are necessary to obtain valid estimates of the effects of marketing activities. For the multiplicative sales response function, Christen et al. (1997) demonstrate that heterogeneity of marketing activities across stores leads to aggregation bias even if marketing effects are homogeneous across stores (i.e. all coefficients are the same).

Heterogeneous effects (e.g. different price coefficients of individual stores) constitute another source of aggregation bias. Krishnamurthi/Raj/Selvam (1990) show that this type of heterogeneity typically leads to aggregation bias even in the case of a linear model. For the multiplicative function, van Garderen/Lee/Pesaran (2000) prove that given heterogeneous effects its coefficients are not identified.

Costs of acquisition and analysis of store-specific data are higher than those of aggregate data. This fact can be explained by several factors. Firstly, data volume is higher and data analysis takes more time. Secondly, marketing managers often have only access to aggregate data, e.g. at the chain level (Hanssens/Parsons/Schultz 2001; Leebleng et al. 2000). If they want a detailed analysis at the store level, they have to outsource it or cooperate with a retail chain.

These factors work no matter whether store-specific data are analyzed by homogeneous sales response models.
Table 1: Descriptive statistics

<table>
<thead>
<tr>
<th>Brands</th>
<th>Mean Sales</th>
<th>Mean Standard deviation</th>
<th>Mean Prices</th>
<th>Mean Standard deviation</th>
<th>Mean relative absolute difference across stores</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55.12</td>
<td>100.76</td>
<td>285.41</td>
<td>33.01</td>
<td>41.44%</td>
</tr>
<tr>
<td>2</td>
<td>195.85</td>
<td>251.50</td>
<td>293.96</td>
<td>57.21</td>
<td>42.21%</td>
</tr>
<tr>
<td>3</td>
<td>264.78</td>
<td>392.41</td>
<td>218.18</td>
<td>38.45</td>
<td>22.07%</td>
</tr>
<tr>
<td>4</td>
<td>290.55</td>
<td>497.38</td>
<td>220.86</td>
<td>42.07</td>
<td>21.33%</td>
</tr>
<tr>
<td>5</td>
<td>99.62</td>
<td>264.19</td>
<td>229.61</td>
<td>35.66</td>
<td>22.13%</td>
</tr>
<tr>
<td>6</td>
<td>48.67</td>
<td>95.08</td>
<td>214.26</td>
<td>31.16</td>
<td>20.55%</td>
</tr>
<tr>
<td>7</td>
<td>56.61</td>
<td>271.32</td>
<td>214.82</td>
<td>41.60</td>
<td>21.74%</td>
</tr>
<tr>
<td>8</td>
<td>307.85</td>
<td>517.74</td>
<td>174.39</td>
<td>41.51</td>
<td>26.43%</td>
</tr>
<tr>
<td>9</td>
<td>53.05</td>
<td>54.79</td>
<td>149.08</td>
<td>8.85</td>
<td>26.95%</td>
</tr>
</tbody>
</table>

Decisions made on the basis of a model which suffers from aggregation bias may lead to lower profits. If, for example, the less valid model overestimates (underestimates) absolute price effects, management sets prices which are lower (higher) than the optimal price. What is important from a managerial point of view is not bias per se, but the effect bias has on expected profit. From this perspective management would act rationally by sticking to a simpler (e.g., aggregate or homogeneous), though biased model, if it leads to more or less the same results with respect to expected profits. A more complex and costly model would only be used if it can be expected to improve decision making, i.e. to result in higher profits.

We follow an approach suggested by Rossi and Allenby (2003) to valuate disaggregate information. Specifically, we compare expected profit implied by the heterogeneous store-specific model to expected profit

1. that could be obtained by using the aggregate model;
2. that could be obtained by using the homogeneous store-specific model.

Substituting the aggregate (homogeneous) model for the homogeneous (heterogeneous) model can be recommended if the expected profit implied by the latter is greater.

2. Sales Response Models Studied

Estimation of sales response models is based on store-level data which were acquired in 81 stores belonging to Dominick’s Finer Foods Chain in Chicago, IL. Between 61 and 88 weeks per store lead to a total of 62,878 obser-
vations. The data analyzed refer to the nine leading brands of the refrigerated orange juice category. Table 1 contains descriptive statistics (means and standard deviations) of sales and prices for these nine brands.

Originally several functional forms of sales response models were considered, namely linear, multiplicative, exponential, semilog (taking logs of prices), logistic, asymmetric logistic and the Gutenberg model using Albach’s specification (Hanssens/Parsons/Schultz 2001; Hruschka 1996). With the exception of the Gutenberg model, these models include as predictors price of the respective brand and average price across all other brands observed in the same store. The Gutenberg model has as predictors own price and price difference to this average price. A heterogeneous version of each of these models with coefficients varying across stores was estimated by an appropriate Markov Chain Monte Carlo method (see section 3).

As we are interested in pricing decisions on the aggregate level, we evaluate these functional forms by a measure based on errors at the chain-level. More specifically, following van Garderen/Lee/Pesaran (2000), performance of functional forms is measured by the sum of squared aggregate errors (SSAE):

$$SSAE_m = \sum_{t=1}^{T} (Q_{mt} - \hat{Q}_{mt})^2.$$  \hspace{1cm} (1)

$Q_{mt}$ denotes the chain-level sales of brand $m$ in week $t$, $\hat{Q}_{mt}$ its value estimated by a model. For heterogeneous models, $\hat{Q}_{mt}$ is determined as the arithmetic mean of estimated chain-level sales values over $S$ sampled coefficient vectors.

Both multiplicative and semilog functions reduce SSAE compared to the linear function with SSAE being 0.87 and 0.90 times that of the linear model, respectively. Performance of the other functional forms is worse, e.g. for the exponential form SSAE equals 1.04, for the logistic form it equals 1.29 times that of the linear model (Table 2). Because of this superiority, the remainder of this paper deals with the multiplicative model only.

Model specifications originally also included coefficients for POS displays of the respective brand and its competitors as well as prices or sales both lagged by one week as possible dynamic effects. The maximum condition number of the matrix of cross products including all these predictors across all brands amounted to 14.27. As this result ruled out multicollinearity problems, heterogeneous store-level models encompassing all coefficients were estimated. This estimation exercise demonstrated that the additional variables mentioned may be eliminated as 95 % credible intervals of their coefficients include zero. The probability that a coefficient lies in the 95 % credible interval given the observed data is at least 95 % (Carlin/Louis 1996). The better known 95 % confidence interval says that 95 % of the confidence intervals computed for a large number of data sets collected the same way as the one analyzed contain the true value. That is why these variables are ignored in the following.

Heterogeneity of pricing activities is measured by the mean relative absolute difference of prices of each brand $m$ in each store from the weekly average $p_{mit}$ across stores. This measure is defined as follows:

$$100/(T \times I) \sum_{m=1}^{M} \sum_{i=1}^{I} \frac{\text{abs}(p_{mit} - p_{mit})}{p_{mit}}$$ with $p_{mit} \equiv 1/\sum_{j=1}^{J} p_{mj}$ \hspace{1cm} (2)

$p_{mit}$ denotes the price of brand $m$ at store $i$ in week $t$, $T$ the number of weeks and $I$ the number of stores.

Van Garderen/Lee/Pesaran (2000) demonstrate that for the multiplicative model aggregation bias increases with the measure defined in (2). As can be seen from the last column of Table 1, the data reflect a rather high degree of heterogeneity of pricing with (rounded) relative differences between 21 % and 42 %.

Model types studied differ both with regards to aggregation level and consideration of heterogeneous effects. Two aggregation levels, chain level and store level, are distinguished (characteristics of model types studied are summarized in Table 3).

Models with store-level sales as dependent variable may be heterogeneous or homogeneous. The expression for a heterogeneous store-level sales model (called heterogeneous model in the following) is:

$$\hat{Q}_{nit} = a_{nit} p_{nit}^{b_{nti}} \prod_{j \in J_n} p_{jti}^{b_{ntj}}.$$  \hspace{1cm} (3)

$J_n$ denotes the index set of competing brands which affect sales of brand $m$ which may be empty. $J_n$ consists of those brands for which the 95 % credible intervals of mean price coefficients of the heterogeneous model do not cover positive and negative values. Note that competitive prices $p_{ji}$ with $j \in J_n$ are observed at the same store $i$ as the price of brand $m$ $p_{mit}$. All coefficients of the heterogeneous model, i.e. constant $a_{nit}$ own price coefficient $b_{nti}$ and coefficients of prices of competing brands $b_{ntj}$, are estimated by a column vector $\beta_{nit}$. Store-specific coefficient vectors $\tilde{\beta}_{nit}$ of stores $i = 1, \ldots, I$ are assumed to be multivariate normally distributed with mean vector $\beta_{nit}$ and covariance matrix $\Sigma_{ni}$.

Put differently, the heterogeneous model can be seen as

<table>
<thead>
<tr>
<th>Functional Form</th>
<th>Sum of Squared Aggregate Errors Relative to Linear Model*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplicative</td>
<td>0.87</td>
</tr>
<tr>
<td>Exponential</td>
<td>1.04</td>
</tr>
<tr>
<td>Semilog</td>
<td>0.90</td>
</tr>
<tr>
<td>Logistic</td>
<td>1.29</td>
</tr>
<tr>
<td>Asymmetric Logistic</td>
<td>1.33</td>
</tr>
<tr>
<td>Gutenberg</td>
<td>1.38</td>
</tr>
</tbody>
</table>

* averages across brands

Table 2: Evaluation of functional forms
recession model whose coefficients follow a continuous distribution (the multivariate normal) and do not depend on variables (e.g. store characteristics).

The continuous distribution of coefficients together with equation (3) constitutes a hierarchical model as the so-called hyperparameters \( \beta_m \) and \( \Sigma_m \) affect the dependent variable through the store-specific coefficient vectors only. It is characteristic of hierarchical models that each store-specific coefficient depends on data from all stores (Carlin/Louis 1996). Therefore coefficients are less noisy and unstable than those based on estimating single store-specific regression models (see e.g. the empirical results obtained by Boatwright/McCulloch/Rossi 1999).

Assuming that coefficients follow a finite mixture distribution would entail an alternative approach. As a rule, empirical applications of finite mixture models in marketing lead to a rather small number of components (e.g., five or six). Therefore finite mixture models are prone to underestimate heterogeneity (Allenby/Rossi 1999). Moreover, for modeling of store-level data the continuous approach is obviously more frequent than the finite mixture approach (examples are Blattberg/George 1991; Boatwright/McCulloch/Rossi 1999; Montgomery 1997; Montgomery/Rossi 1999). These aspects justify our decision to estimate models with continuous distributions of coefficients.

Homogeneous models are characterized by equal coefficients across stores. The homogeneous store-level sales model (called homogeneous model in the following) for brand \( m \) is:

\[
Q_{mt}^\text{hom} = a_{m}^\text{hom} \prod_{j \in L} p_{jt}^{b_{m,j}^\text{hom}}. \tag{4}
\]

\( a_{m}^\text{hom} \) symbolizes the constant term, \( b_{m,j}^\text{hom} \) the exponent of price of brand \( j \) for the homogeneous sales response model of brand \( m \).

Chain-level sales \( Q_{mt} \) of brand \( m \) in week \( t \) are equal to the sum of its store-level sales across \( I \) stores:

\[
Q_{mt} = \sum_{i} Q_{mit}. \tag{5}
\]

Chain-level price \( p_{mt} \) of brand \( m \) in week \( t \) corresponds to the arithmetic mean of store-level prices across \( I \) stores. The chain-level sales response model (called aggregate model in the following) for brand \( m \) may be written as:

\[
Q_{mt}^\text{agg} = a_{m}^\text{agg} \prod_{j \in L} p_{jt}^{b_{m,j}^\text{agg}}. \tag{6}
\]

\( a_{m}^\text{agg} \) symbolizes the constant term, \( b_{m,j}^\text{agg} \) the exponent of price of brand \( j \) for the aggregate sales response model of brand \( m \).

Let vectors \( p_{m}^{\text{agg}} \) and \( p_{m}^{\text{hom}} \) contain the parameters for the aggregate and homogeneous models, respectively, vectors \( p_{m,i}^{\text{agg}} \) and \( p_{m,i}^{\text{hom}} \) prices and average (across stores) prices of the competitors of brand \( m \), respectively. This allows introduction of a more compact notation for the three types of sales response models studied:

\[
Q_{mt}^\text{agg} = \hat{Q}_{mt}^\text{agg}(p_{m}^{\text{agg}}, p_{m}, p_{c}^{\text{agg}}), \tag{7}
\]

\[
Q_{mt}^\text{hom} = \hat{Q}_{mt}^\text{hom}(p_{m}^{\text{hom}}, p_{m}, p_{c}^{\text{hom}}), \tag{8}
\]

\[
Q_{mt}^\text{agg} = \hat{Q}_{mt}^\text{agg}(p_{m}^{\text{agg}}, p_{m}, p_{c}). \tag{9}
\]

Chain-level elasticities are important as they determine optimal pricing decisions on the chain level. As is well known from the multiplicative model, price elasticities at the chain level are equal to the price coefficient \( b_{m,j}^\text{agg} \) and \( b_{m,j}^\text{hom} \) for the aggregate and homogeneous sales response models, respectively. But this property is no longer true for the heterogeneous store-level model because chain-level sales are equal to the sum of store-level sales each estimated by an individual multiplicative sales response model. Therefore chain-level price elasticity \( e_{m} \) according to the heterogeneous model corresponds to the average of store-specific price coefficients weighted by the ratio of store-specific sales and chain-level sales:

\[
e_{m} = \sum b_{m,i} \frac{\hat{Q}_{mi}}{\sum Q_{mi}} = \sum b_{m,i} \frac{\hat{Q}_{mi}}{Q_{mi}}. \tag{10}
\]

3. Estimation

Estimation of the types of the multiplicative models defined in equations (3), (4) and (6) works on their log-transformations:

\[
\log(\hat{Q}_{mt}^\text{agg}) = \log(a_{m}) + b_{m,j}^\text{agg} \log(p_{jt}) + \sum_{j \in L} b_{m,j}^\text{agg} \log(p_{jt}). \tag{11}
\]

\[
\log(\hat{Q}_{mt}^\text{hom}) = \log(a_{m}^\text{hom}) + b_{m,j}^\text{hom} \log(p_{jt}) + \sum_{j \in L} b_{m,j}^\text{hom} \log(p_{jt}). \tag{12}
\]

\[
\log(\hat{Q}_{mt}^\text{agg}) = \log(a_{m}^\text{agg}) + b_{m,j}^\text{agg} \log(p_{jt}) + \sum_{j \in L} b_{m,j}^\text{agg} \log(p_{jt}). \tag{13}
\]

Residuals of these log-transformed models are assumed to be log-normally distributed with location parameter equal to zero and constant error variance.

Aggregate and homogeneous sales response models are estimated by ordinary least squares (OLS). Because of

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**Table 3: Characteristics of model type studied**

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Sales</th>
<th>Prices</th>
<th>Coefficients Across Stores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate</td>
<td>sum across stores</td>
<td>average across stores</td>
<td>--</td>
</tr>
<tr>
<td>Homogeneous</td>
<td>store-level</td>
<td>store-level</td>
<td>are equal</td>
</tr>
<tr>
<td>Heterogeneous</td>
<td>store-level</td>
<td>store-level</td>
<td>are different</td>
</tr>
</tbody>
</table>

---

**Equations**

\[ Q_{mt} = \sum_{i} Q_{mit} \]

\[ Q_{mt}^\text{agg} = a_{m}^\text{agg} \prod_{j \in L} p_{jt}^{b_{m,j}^\text{agg}} \]

\[ Q_{mt}^\text{hom} = a_{m}^\text{hom} \prod_{j \in L} p_{jt}^{b_{m,j}^\text{hom}} \]

---

**References**

- Allenby/Rossi 1999
- Allenby/Rossi 1997
- Montgomery/Rossi 1999
- Montgomery 1997
- Boatwright/McCulloch/Rossi 1999
- Blattberg/George 1991
- Carlin/Louis 1996
their hierarchical form which was explained in the previous section heterogeneous models cannot be estimated by OLS. Therefore a Markov Chain Monte Carlo (MCMC) simulation technique is used.

The MCMC technique generates random samples of parameters from their joint posterior distribution after convergence to stationarity. In principle, samples could be drawn directly from the joint posterior distribution. But this approach would be very slow and MCMC techniques like Gibbs sampling and Metropolis-Hastings algorithms offer a less time consuming alternative (Gelman et al. 1995). The MCMC technique used is based on a method developed by Train (2001, 2003). Statistics (e.g. means, percentiles etc.) of values sampled by this method converge to their population values.

Gibbs sampling draws sequentially from distributions of each parameter conditional on the other parameters and eventually provides draws from the joint distribution. Gibbs sampling is particularly attractive if draws from a conditional distribution can be quickly obtained. In the case of the heterogeneous multiplicative model, this property applies to error variance, mean coefficients and their covariance matrix. The relevant conditional posterior distributions are (Chib/Greenberg 1995; Gelman et al. 1995):

- Gamma for the error precision, i.e. the inverse of the error variance;
- multivariate normal for the mean coefficients;
- inverted Wishart for the covariance matrix of mean coefficients.

In addition, a Metropolis-Hasting algorithm samples store-specific coefficients using a candidate generating multivariate normal distribution. This algorithm probabilistically accepts candidate values depending on the ratio of posterior probabilities of old and candidate values (Gelman et al. 1995).

To summarize, in each of several iterations the MCMC technique consists of four substeps:

1. a Metropolis-Hastings algorithm which samples store specific coefficients \( \beta_{\text{sm}} \) (Chib/Greenberg 1995; Gelman et al. 1995).
2. a Gibbs sampler which draws the error precision from the conditional Gamma distribution whose parameters depend on the sum of squared errors and number of observations.
3. a Gibbs sampler which draws from the conditional multivariate normal distribution of mean coefficients \( \beta_{\text{sm}} \) given store-specific coefficients \( \beta_{\text{mi}} \) and their covariance matrix \( \Sigma_{\text{m}} \).
4. a Gibbs sampler which draws from the conditional inverted Wishart distribution of the covariance matrix \( \Sigma_{\text{m}} \) given store-specific and mean coefficients.

Trace plots and autocorrelations with a maximum lag of 50 iterations for mean coefficients serve to assess whether the MCMC technique does not converge (Kass et al. 1998). In our empirical study, the MCMC technique runs 4000 iterations for each model, the last 2000 of which give the sampled coefficients. Trace plots show that for most models mean coefficients become stable at less than 2000 iterations. Autocorrelations also do not indicate nonconvergence.

Endpoints of 95% credible intervals of (functions of) coefficients are given by 2.5 and 97.5 percentiles of sampled values (Carlin/Louis 1996). The posterior probability that a parameter is greater than zero (less than zero) (Carlin/Louis 1996) equals the relative frequency of such values.

4. Statistical Model Evaluation

One statistical evaluation criterion used in this paper is SSAAE which was introduced in section 2 by equation (1). If the true underlying disaggregate model is nonlinear, aggregate errors diminish if a model becomes more detailed (Foekens/Leeflang/Wittink 1994). Therefore store-level models are expected to be more accurate than chain-level models, heterogeneous models to be more accurate than homogeneous models.

As residuals of sales equations are log-normally distributed, to obtain estimated sales values the antilog of the output value of the relevant equation (11), (12) or (13) is multiplied by \( \exp(1/2\sigma_u^2) \) with \( \sigma_u^2 \) denoting the error variance.

To determine SSAAE, estimated sales values \( \hat{Q}_{\text{mt}} \) are needed. Their computation depends on the type of model considered and is described in the following. For aggregate models, estimated sales are simply equal to the estimated value of the dependent variable \( \hat{Q}_{\text{mt}}^{\text{agg}} \).

For homogeneous and heterogeneous store-level models, chain-level sales \( \hat{Q}_{\text{mt}} \) are obtained by summing estimated store-level sales across stores:

\[
\hat{Q}_{\text{mt}} = \sum_{i=1}^{I} \hat{Q}_{\text{mi}}^{\text{hom}} \tag{14}
\]

or

\[
\hat{Q}_{\text{mt}} = \sum_{i=1}^{I} \hat{Q}_{\text{mi}}^{\text{het}} . \tag{15}
\]

Whereas for homogeneous models store-level sales \( \hat{Q}_{\text{mt}}^{\text{hom}} \) equal the estimated value of the dependent variable, for heterogeneous models store-level sales \( \hat{Q}_{\text{mt}}^{\text{het}} \) are estimated as the arithmetic mean of sales values computed using \( S \) samples of coefficient \( \beta_{\text{sm}}, s = 1, ..., S \):

\[
\hat{Q}_{\text{mt}}^{\text{het}} = 1/S \sum_{s=1}^{S} \hat{Q}_m^{\text{het}} (\beta_{\text{mis}}, \mu_m, \beta_{\text{ms}}). \tag{16}
\]

The other statistical criterion used to evaluate models refers to biases of chain-level elasticities. Aggregate or homogeneous models are judged to be biased if their chain-level elasticities differ from those of the heterogeneous.
neous model. To compare chain-level elasticities we determine the 2.5, 50 and 97.5 percentiles of sampled values of elasticities at the average price of brand \( m \) (\( \bar{p}_m \)) for each model. Each sampled value in turn is an average across all \( T \) observed prices of competitors. 2.5 and 97.5 percentiles define the 95 \% credible set of elasticities. The aggregate (homogeneous) model differs from the heterogeneous model with respect to chain-level elasticities if their respective 95 \% credible intervals do not overlap.

For the heterogeneous model chain-level price elasticities at average prices are estimated using \( S \) samples of store-specific coefficient \( \beta_{mis} \), \( s = 1, \ldots, S \). \( \beta_{hom} \) denotes sample \( s \) of the price coefficient of brand \( m \) for store \( i \) generated by the MCMC technique described above. Because of expression (10) \( S \) samples of elasticity is:

\[
\frac{\sum_i Q_{mis} - \sum_i \bar{Q}_{mis}}{\sum_i \bar{Q}_{mis}} = \frac{1}{T} \sum_i \frac{Q_{mis}(\beta_{mis} \cdot \bar{p}_m \cdot \bar{p}_m)}{\sum_i \bar{Q}_{mis}(\beta_{hom} \cdot \bar{p}_m \cdot \bar{p}_m)}.
\]

(17)

For aggregate and homogeneous models, samples of elasticities are simply obtained by sampling coefficients, as the chain-level price elasticity equals the price coefficient \( \beta_{mis} \) or \( \beta_{hom} \). For these models, samples of coefficients are generated by an iterative technique which consists of two substeps. In the first substep, it draws a random number \( z \) from a \( \chi^2 \)-distribution with \( v = N \cdot k \) degrees of freedom (\( N \) number of observations, \( k \) number of coefficients). In the second substep, it draws a sample of coefficients from the conditional multivariate normal distribution which has OLS coefficients and their covariance multiplied by \( v / z \) as parameters (Tanner 1996).

5. Managerial Model Evaluation

It is assumed that management wants to maximize chain-level profit \( \Pi_m \) of brand \( m \) in any week \( t \) given constant variable unit costs \( k_m \):

\[
\Pi_m = (p_m - k_m)Q_{mis}.
\]

(18)

The only decision variable is the chain-level price of brand \( m \), \( p_m \), whose optimal value is constant across weeks as sales response models do not include dynamic effects. Management expects that competitors set prices at values with frequencies observed in the past (this solution concept is known as Fictitious Play (Brown 1951)). Therefore profits are averaged across \( t = 1, \ldots, T \) observed prices of competitive brands. As information on variable unit costs is not available they are approximated by rewriting the optimality condition for price. Starting from the objective function (18), one obtains the following first-order condition for profit-maximal price \( p_m \):

\[
\frac{\partial \Pi_m}{\partial p_m} = (p_m - k_m) \frac{\partial Q_{mis}}{\partial p_m} + Q_{mis} = 0.
\]

(19)

By introducing price elasticity \( e_m = \frac{\partial Q_{mis}}{\partial p_m} \cdot \frac{p_m}{Q_{mis}} \) this condition can also be written in the following way:

\[
(p_m - k_m)e_m \frac{Q_{mis}}{p_m} + Q_{mis} = 0.
\]

(20)

Dividing by \( Q_{mis} \) and moving \( p_m \) to the left-hand side leads to:

\[
p_m = \frac{e_m}{1 + e_m}k_m.
\]

(21)

Rewriting optimality condition (21) finally gives:

\[
k_m = \frac{1 + e_m}{e_m}p_m.
\]

(22)

To approximate \( k_m \), we insert into expression (22) the average price of brand \( m \) for \( p_m \) and its chain-level elasticity derived on the basis of the most accurate model (\( \beta_{hom} \)) for \( e_m \).

Computations use an evenly spaced grid of 100 price values \( p_{mji} \) \( j = 1, \ldots, 100 \), over the interval defined by observed minimal and maximal prices for each brand \( m \). For each price \( p_{mji} \) the expected profit \( E[\Pi_m^{het}(p_{mji})] \) when using the heterogeneous model is obtained as the arithmetic mean across \( S \) samples of coefficients summing over stores and averaging across observations:

\[
E[\Pi_m^{het}(p_{mji})] = \frac{1}{S} \sum_i \Pi_m^{het}(p_{mji}, s),
\]

(23)

\[
= \frac{1}{S} \sum \frac{1}{T} \sum Q_{mis}^{het}(\beta_{mis} \cdot p_{mji} \cdot \bar{p}_m)(p_{mji} - k_m).
\]

The optimal price \( p_{mji}^{opt} \) is the price which leads to the highest expected profit:

\[
p_{mji}^{opt} = \arg \max_{p_{mji}} E[\Pi_m^{het}(p_{mji})].
\]

(24)

Management sets prices based on either the aggregate or the homogeneous models trying to maximize expected profit. If management uses the aggregate model, prices are set with respect to the following expression for expected profit:

\[
E[\Pi_m^{agg}(p_{mji})] = \frac{1}{S} \sum_i Q_{mis}^{agg}(\beta_{agg} \cdot \bar{p}_m)(p_{mji} - k_m).
\]

(25)

Management considers the following price \( p_{mji}^{agg} \) as “optimal” which maximizes expected profit on the basis of the aggregate model:

\[
p_{mji}^{agg} = \arg \max_{p_{mji}} E[\Pi_m^{agg}(p_{mji})].
\]

(26)

As a rule, the heterogeneous model is more accurate in forecasting expected sales and profits than the aggregate model. Therefore profits are higher if management would use the heterogeneous model to set prices. We want to know how much profit management gives up by using the simpler aggregate model. To this end, price \( p_{mji}^{agg} \) is plugged into the expression for the heterogeneous model:

\[
E[\Pi_m^{het}(p_{mji}^{agg})] = \frac{1}{S} \sum_i \Pi_m^{het}(p_{mji}^{agg}, s),
\]

(27)

\[
= \frac{1}{S} \sum \frac{1}{T} \sum Q_{mis}^{het}(\beta_{mis} \cdot p_{mji}^{agg} \cdot \bar{p}_m)(p_{mji}^{agg} - k_m).
\]

In other words, \( E[\Pi_m^{agg}(p_{mji}^{agg})] \) denotes the expected profit when using the aggregate model.

If management uses the homogeneous model, prices are set with respect to the following expression for expected profit:
Management considers the following price $p_{m\text{hom}}$ as “optimal” which maximizes expected profit on the basis of the homogeneous model:

$$p_{m\text{hom}} = \arg\max_{p_{m\text{hom}}} E[\Pi_{m\text{hom}}(p_{m\text{hom}}, s)].$$

(29)

As a rule, the heterogeneous model is more accurate than the homogeneous model in forecasting expected sales and profits. Therefore, profits are higher if management would use the heterogeneous model to set prices.

We want to know how much profit management gives up by using the simpler homogeneous model. To this end, price $p_{m\text{hom}}$ is plugged into the expression for the heterogeneous model:

$$E[\Pi_{m\text{het}}(p_{m\text{hom}})] = 1/S \sum_{t=1}^{S} \sum_{j=1}^{T} \sum_{i=1}^{N} \Pi_{m\text{het}}(p_{m\text{ag}}^t, p_{m\text{het}}^t, p_{m\text{ag}}^t, p_{m\text{ag}}^t)(p_{m\text{het}} - k_m).$$

(30)

In other words, $E[\Pi_{m\text{het}}(p_{m\text{hom}})]$ denotes the expected profit when using the homogeneous model.

We want to infer whether profit obtainable by using the homogeneous (heterogeneous) model is higher than profit for the aggregate (homogeneous) model. To this end, 95 % credible intervals of expected profits are estimated by the 2.5 and 97.5 percentiles of $S$ samples, i.e. $\Pi_{m\text{hom}}(p_{m\text{ag}}, s)$, $\Pi_{m\text{het}}(p_{m\text{ag}}, s)$ and $\Pi_{m\text{het}}(p_{m\text{ag}}, s)$ with $s = 1, \ldots, S$. In this sense, profit using the homogeneous (heterogeneous) model is classified to be higher if its 95 % credible interval does not overlap with that of the aggregate (homogeneous) model.

Additionally, we define an index of preferability of the homogeneous (heterogeneous) model as percentual difference of expected profits with respect to the aggregate (homogeneous) model:

$$100 \frac{E[\Pi_{m\text{het}}(p_{m\text{hom}})] - E[\Pi_{m\text{het}}(p_{m\text{ag}})]}{E[\Pi_{m\text{het}}(p_{m\text{hom}})]}$$

(32)

or

$$100 \frac{E[\Pi_{m\text{het}}(p_{m\text{het}})] - E[\Pi_{m\text{het}}(p_{m\text{ag}})]}{E[\Pi_{m\text{het}}(p_{m\text{het}})]}.$$ 

(33)

6. Empirical Study

6.1. Statistical Evaluation

Use of store-specific data leads to a drastic reduction of aggregate error (Table 4). SSAE of aggregate models is on average more than nine times as high as SSAE of heterogeneous models. SSAE of the homogeneous model is on average 1.63 as high as SSAE of the heterogeneous model. Though less dramatic, this difference is still important from a statistical point of view.

To summarize, the heterogeneous model turns out to be the most accurate, as expected. Table 5 contains mean coefficients (estimated by averaging across samples).

This table also provides posterior probabilities that the coefficient of the respective brand’s own price is less than zero and that each of the other coefficients is greater than zero. Note the very high posterior probabilities of the nine coefficients of each brand’s own price.

**Table 4: Sum of squared aggregate errors relative to the heterogeneous model**

<table>
<thead>
<tr>
<th>Brand</th>
<th>Aggregate</th>
<th>Homogeneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.41</td>
<td>1.76</td>
</tr>
<tr>
<td>2</td>
<td>17.46</td>
<td>1.91</td>
</tr>
<tr>
<td>3</td>
<td>28.63</td>
<td>1.34</td>
</tr>
<tr>
<td>4</td>
<td>37.4</td>
<td>1.19</td>
</tr>
<tr>
<td>5</td>
<td>19.79</td>
<td>1.64</td>
</tr>
<tr>
<td>6</td>
<td>6.57</td>
<td>1.74</td>
</tr>
<tr>
<td>7</td>
<td>19.2</td>
<td>1.24</td>
</tr>
<tr>
<td>8</td>
<td>11.94</td>
<td>1.50</td>
</tr>
<tr>
<td>9</td>
<td>6.10</td>
<td>2.32</td>
</tr>
</tbody>
</table>

Average: 19.39, 1.63

Christen et al. (1997) report mean elasticities and show that they differ across model types. For our data, median price elasticities (i.e. 50 % percentiles) at the average price of each brand averaged across observed competitors’ prices similarly differ greatly between aggregate and homogeneous models for at least 7 out of 9 brands (Table 6). Differences between homogeneous and heterogeneous models turn out to be much smaller. Of course, such comparisons ignore the uncertainty of elasticity estimates.

**Table 7** gives 95 % credible intervals of chain-level price elasticities at the average price of each brand averaged across observed competitors’ prices. Note that intervals of the aggregate model are much wider, reflecting higher uncertainty of this model. With only one exception (aggregate model, brand 8), intervals show that demand is price elastic.

Despite rather high heterogeneity of prices across stores, chain-level elasticities differ between the aggregate and both the homogeneous model and the heterogeneous model for only three out of nine brands. Evidence for bias of the aggregate model is found for brands 5 and 9, as 95 % credible intervals of chain-level elasticities do not overlap with those for the heterogeneous model.

95 % intervals for the homogeneous and heterogeneous
models for brands 5 and 7 do not overlap and therefore indicate bias of the homogeneous model.

6.2. Managerial Evaluation

For three out of nine brands (i.e. brands 6, 8 and 9), 95 % credible intervals of profits do not overlap between aggregate and heterogeneous models. This means that for these brands pricing based on the aggregate model leads to lower expected profits with probability of 95 % given the data analyzed. For brands 6, 8 and 9, relative differences of profits obtained for the aggregate and the heterogeneous models are greater than 7 % (Table 8). For brand 8, the relative difference of expected profits even amounts to more than 36 %. Interestingly, for the remaining six brands, credible intervals show that pricing on the basis of the aggregate model is not worse (Table 8).

To gain more insight into these results, we look at $\varepsilon_p/(1 + \varepsilon_m)$, the multiplier of marginal costs in optimality condition (21). We compute multipliers for each brand at median elasticities which the aggregate and the homogeneous models imply (Table 6). The last column of Table 8 shows that relative differences of these multipliers between aggregate and homogeneous models take their highest values for brands 6, 8 and 9. In other words, higher differences of cost multipliers, which are nonlinear functions of elasticities, are associated with higher differences of profits.

For all nine brands, decisions based on the more complex heterogeneous model do not achieve higher expected profits at a probability of 95 % given the data analyzed here. Percentual differences of expected profits between the homogeneous and the heterogeneous models are very low.

### 7. Conclusions

The types of sales response models studied here differ with regard to two aspects. Firstly, they are based on
either disaggregate or linearly aggregated data. Secondly, they either preclude or allow heterogeneous effects of marketing instruments across stores. These model types were evaluated both from a statistical and a managerial point of view.

Aggregate (chain-level) models perform worst as aggregate error and uncertainty about coefficients and price elasticities are highest. Chain-level elasticities differ from those inferred on the basis of the store-level models for three out of nine brands.

Expected profits for prices set at the aggregate (chain-) level serve as managerial criterion to evaluate models. For three out of nine brands, expected profits increase by more than 7 % if store-level data are analyzed. Our results show that these profit differences can be traced back to relative differences of cost multipliers appearing in optimality conditions for prices. These multipliers are nonlinear functions of elasticities.

No difference is found between homogeneous and heterogeneous models. Whereas a more detailed aggregation level turns out to be relevant, consideration of heterogeneity seems not to improve decision-making at the chain level.

It must be emphasized that the following recommendations are tentative, being based on the analysis of one data set only. Further studies are necessary to assess their generalizability. Given the potential to achieve higher profits for some of the brands, analysis of store-level data in place of aggregate data seems to be the advisable option. In our study, the homogeneous model implies profits that do not differ from those obtainable by using the heterogeneous model. As the homogeneous model is easier to use and less costly, it can be considered as a preferable alternative from a managerial point of view if decisions at the aggregate (chain) level are at stake.

References


Table 8: Differences between aggregate and heterogeneous models

<table>
<thead>
<tr>
<th>Brand</th>
<th>Overlap of 95 % credible intervals of profits</th>
<th>Percentual difference of Marginal cost multipliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>yes</td>
<td>1.30</td>
</tr>
<tr>
<td>2</td>
<td>yes</td>
<td>0.96</td>
</tr>
<tr>
<td>3</td>
<td>yes</td>
<td>0.55</td>
</tr>
<tr>
<td>4</td>
<td>yes</td>
<td>1.38</td>
</tr>
<tr>
<td>5</td>
<td>yes</td>
<td>1.03</td>
</tr>
<tr>
<td>6</td>
<td>no</td>
<td>7.21</td>
</tr>
<tr>
<td>7</td>
<td>yes</td>
<td>4.25</td>
</tr>
<tr>
<td>8</td>
<td>no</td>
<td>36.25</td>
</tr>
<tr>
<td>9</td>
<td>no</td>
<td>7.13</td>
</tr>
</tbody>
</table>