When Five is a Crowd in the Market Share Attraction Model
The Dynamic Stability of Competition

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In this paper we use a very simple model of competition to show how and why five or more competitors vying for profits in an early-stage market might create turbulence in marketing spending. If the turbulence leads to a shakeout (exit of competitors), the same decision rules that caused the turbulence with five or more competitors can then lead the remaining four or fewer competitors to an equilibrium. We believe this is consistent with the notion that non-equilibrium behavior may be part of an evolutionary economy; a market with convergent properties might emerge from one that does not have those properties.

We use a model in which buyer reactions follow the market share attraction model (MSA), and sellers may be viewed as following either of two assumptions to optimize their own profits. The first is that sellers use sophisticated technique to measure current customer responses to a range of marketing budgets and choose the current profit-maximizing budget for the next period. A second, completely equivalent, logic is the traditional Cournot assumption: the firm adjusts its spending under an assumption that competitors will not change last-period budgets.

Application of either logic will lead to dynamic instability only when the number of competitors exceeds a threshold level of four. This study provides an analytical explanation for the emergence of this instability and demonstrates the relationship between this system of competing firms and the standard logistic map. We also analyze market conditions that might affect the existence or level of the convergence properties. These include partial adjustment to optimal spending, exogenous growth (independent of marketing spending), and endogenous growth, non-linear marketing attractiveness to spending relationships, carryover effects, and differences in seller marketing efficiencies.

Keywords
Market share attraction model, marketing spending, Nash equilibrium, industry shakeouts, logistic map

Not only in (biological) research, but also in the everyday world of politics and economics, we would all be better off if more people realized that simple nonlinear systems do not necessarily possess simple dynamical properties.

May (1986)
1. Introduction

The Market Share Attraction (MSA) model is the simplest representation of buyer reaction to competition among sellers that we know. In the MSA model, the basic equation for market share is

\[ s_i = \frac{A_i}{\sum A_j}, \]

(1)

where \( s_i \) denotes share and \( A_i \) the attraction of firm \( i \) \((i = 1, ..., m)\). Farris et al. (1998) observed that simulated firms myopically optimizing spending in a MSA market quickly reached an equilibrium spending level when the number of firms was small, but exhibited turbulence and "chaos" when the number of firms in the industry reached a critical threshold of five. The purpose of this paper is to explain and explore further this "five is a crowd" phenomenon.

First we will use simulation to reproduce the phenomenon. Then with a slight modification of the model to allow for differences in spending effectiveness across firms, we observe shakeouts in the simulated industries. This simple two-parameter model, although entirely deterministic, produces "unpredictable" shakeouts, where the number of surviving firms can be anything from 0 to 4, but never five or more. Again using simulation, we introduce a parameter \( k \) representing the speed of adjustment. For carefully selected combinations of \( k \) and \( m \) we observe a new phenomenon, 2 and 4 period cycles.

Next we offer an analytical explanation of the phenomenon. Using simple techniques from dynamical system analysis, we show that the recursive equation governing industry spending is similar to the standard logistic map, the product of an equation used to model biological competition. The paper illustrates what we consider to be an interesting property of a very simple model of myopically profit-maximizing firms in a MSA market, that five or more equal competitors generate turbulent competition. The paper illustrates what we consider to be an interesting property of a very simple model of myopically profit-maximizing firms in a MSA market.

Finally, we offer some speculation on the implications of all this. Dismiss it? Believe it totally and literally? Or somewhere in between? Intuitively, we don't believe the result would be nearly as interesting if 200, 25, or 1.3 were a "crowd." The finding that "five is a crowd" corresponds to a common belief that industries often support, and we relax both of these assumptions and explore firm differences.

2. Five is a Crowd

Because our purpose is to explore the basic nature of marketing competition, we keep things as simple as possible. We assume that attraction equals marketing effort, and marketing effort equals marketing spending. Thus we ignore the question of how a firm should allocate its spending across mix elements, but we will have more to say about multiple mix elements in later sections. We emphasize, however, that this single mix element is meant to capture all non-price firm investments in the market: (a) R&D for product development, (b) sales force expenditures, (c) advertising, (d) consumer promotion, (e) trade promotion, (f) inventory, and (g) market research to improve efficiency of these efforts. We also initially assume all firms are equally effective in spending their marketing dollars, and hence the attraction achieved is proportional to dollars spent. In later sections we relax both of these assumptions and explore firm differences in marketing effectiveness and nonlinear attraction/spending relationships.

2.1. The Base Case Model

A market consists of buyers and sellers. In the model, sellers spend marketing resources to attract buyers. Buyers may also be called "consumers" or "customers" and sellers may be called "firms." It is the interaction of buyers and sellers (consumers and firms) that determine "market share" and the subsequent profitability of the competing sellers.

We have the following assumptions in the Base Case Model:

1. We have \( m \) symmetric sellers (i.e. firms);
2. The total market contribution, \( M \), is constant;
3. The Market Share Attraction Model for firm \( i \)'s share at time \( t \):

\[ s_{i,t} = \frac{A_{i,t}}{\sum A_{j,t}}; \]

(2)

4. Attraction is directly proportional to marketing spending, \( x_{i,t} \): \( A_{i,t} = x_{i,t}^{\beta}; \)

(3)

5. We have myopic profit-maximizing firms.
Firm shares defined in assumption 3 refer to shares of MC, the total dollar contribution. Firms use marketing spending to compete for shares of a fixed amount of dollar contribution. (Firms with equal per-unit contributions competing for shares of a fixed number of units is a special case.) Notice also that assumptions 3 and 4 describe buyer reactions to firm spending each and every period. Buyers move swiftly and decisively to allocate current-period share based only on current period spending. (Later in the paper we will relax this assumption and consider carry-over effects.)

In general, assumption 5 means that firms first find the spending amount that maximizes their profit under current market conditions. The current market conditions include the spending of all firms and the buyers’ reactions to that spending. Once the myopic optimal spending amount is found, firms spend that amount in the upcoming period. Thus, under assumption 5 firms spend in period t+1 the amount that would have maximized profit in period t. To be more specific, let \(NP_{i,t}\) represent the net profit to firm \(i\) in period \(t\). Given the fixed market size (assumption 2) and a single spending element, we can write

\[
NP_{i,t} = MC \times s_{i,t} - X_{i,t}.
\]

Under assumptions 3 and 4, net profit can be written as a function of the spending of firm \(i\) and the spending of all other firms

\[
NP_{i,t} = MC \times X_{i,t}/(X_{i,t} + X_{i,t}^*) - X_{i,t},
\]

where \(X_{i,t}^*\) represents the total spending in period \(t\) of all firms other than \(i\). The spending amount which would have maximized firm \(i\) profits in period \(t\) is given as

\[
X_{i,t}^* = \sqrt{MC \times X_{i,t}^* - X_{i,t}}.
\]

Under assumption 5, \(X_{i,t+1}\) will equal the \(X_{i,t}^*\) given above and therefore

\[
X_{i,t+1} = \sqrt{MC \times X_{i,t}^* - X_{i,t}}.
\]

This same behavior is also consistent with what economists call the Cournot adjustment process. While “economists have come to believe that the Cournot assumption is quite unrealistic when applied to pricing decisions,” Scherer (1970, p. 336), Schmalensee (1976) gives several reasons why it may be plausible for marketing spending decisions.

Equation (7) is the recursive equation (map) describing the evolution of seller spending. For \(m\) symmetric firms, the equilibrium, \(X_{NE}\), is the point where \(X_{i,t+1}\) equals \(X_{i,t}\) for all firms.

\[
X_{NE} = \sqrt{MC \times (m - 1)X_{NE}^2 - (m - 1)X_{NE}}, \quad (8)
\]

\[
X_{NE} = MC \times (m - 1)/m^2. \quad (9)
\]

This Nash Equilibrium (NE) was first reported by Mills (1961), and Case (1979, pp. 133-137) derived equations similar to (8) and (9) to describe a situation he called the auto dealer’s game.

### 2.2. Numerical Analysis for \(m = 2\) to \(5\)

To illustrate the dynamic properties of this model, we first simulate the behavior of \(m\) firms for selected values of \(m\) starting all \(m\) firms at identical spending levels in period 0. (In Section 4.1 we explore what happens if the firms start out at different spending levels.) The total industry dollar contribution \(MC\) was fixed at \(100\), an arbitrary amount. At the beginning of each period of the discrete-time simulation, the \(m\) firms optimized their spending based on the market conditions at the end of the previous period.

Figure 1A through 1D illustrate the main results from this simulation study. Each of these figures graphs the time series of total industry spending of the \(m\) profit-maximizing firms. The surprise (to us) was that total industry spending showed stable patterns for two-, three-, and four-firm industries, but chaos reigned when a fifth firm was added. How and why does this happen? Notice also that with fixed industry size and costs, industry total profits are simply \(100\) minus the industry spending amounts shown in these graphs. Thus, two-through four-firm industries showed stable profit patterns while the fifth firm led to instability.

Let us comment in more detail. With two or three competitors, the industry moves quickly and steadily to equilibrium. With four competitors, industry spending oscillates around the eventual equilibrium level. Notice, however, that the equilibrium spending level depends on \(m\) as given in (9). The major difference between the patterns for \(m = 3\) and \(4\) is the rate at which the firms converge to equilibrium - very quickly for \(m = 3\) and quite slowly for \(m = 4\). These patterns appear to be unaffected by the starting value.

The tendency toward an equilibrium spending level changes most dramatically when we move to five firms. Industry spending appears to be chaotic, showing almost no pattern or predictability. While an equilibrium spending level exists, it appears the simple profit-maximizing behavior that leads 2, 3, or 4 competitors to equilibrium
Figure 1: Industry spending for 2, 3, 4, and 5-firm industries

has radically different implications for 5 competitors. Even if five competitors get close to the NE (as they did in period 21), they do not stay there for very long. Indeed, this model appears balanced at the edge of chaos.

Six or more firms make the simulated spending even more erratic. Whereas 2-5 firms stay “in bounds” – yielding feasible values for spending, six or more do not. Within 2-4 periods, depending on the starting values, simulations of six or more firms invariably hit a period where each seller spends more than $\text{MC}/(m-1)$. When that happens, the sellers find themselves in a market where there is no way to make a profit. The “optimal” spending calculated from (6) is negative and therefore infeasible. The simulations terminate. If we are willing to interpret this as sellers exiting the market, we have the beginnings of a simple model of industry shakeout.

2.3. A (Very) Simple Model of Industry Shakeouts

Our model for shakeouts starts with the assumption that sellers have differential levels of marketing effectiveness. We operationalize this differential marketing effectiveness as differences in the firm attraction generated per marketing dollar spent, such that

$$A_{i,t} = a_i X_{i,t},$$

(10)

where $a_i = 1$ for firm 1, $1+\delta$ for firm 2, $1+(2\delta)$ for firm 3, ..., $1+(m-1)\delta$ for firm $m$. The parameter $\delta$ is a measure of the heterogeneity of marketing effectiveness.

As was the case with more than five firms, there will be periods when the least efficient firms find themselves with no way to make a profit. In those situations, their calculated optimal spending is negative. If we assume sellers permanently exit the industry in such situations, we have a very simple model of industry shakeouts [1]. By permanent exit we mean the seller spends zero in all subsequent periods.

We know that sellers will exit in order from least efficient to most efficient. And we will use simulation to explore some of the things we do not know. How long will it take for the least efficient firm to exit? Will sellers exit one at a time or in bunches? How many firms will survive? Will the industry always reach equilibrium?

We start each simulation with 10 firms. Although initial spending levels were varied among simulation runs, within a simulation run the same initial spending level was used for each firm. The parameter $\delta$ was varied from 0.01 to 0.50. We report selected values in this range. These values correspond to a range of relative marketing effectiveness from the lowest to the highest effectiveness level of 1.09:1 for the narrowest range to 5.5:1 for the widest range.

Each simulation was run for 100 periods and the number of surviving firms evaluated at $t = 10$ and $t = 100$. In every case the number of competitors had stabilized by $t = 10$. Table 1 gives the number of surviving sellers as a function of $\delta$ and initial budget. Figure 2 also plots the
time series of industry spending and number of firms for three selected simulation runs.

The results tell us that in our model industry shakeouts happen relatively quickly (by period ten in all our simulations) and in multiple stages. Sellers can drop out one at a time or in bunches. Our simulated industries always reached equilibrium (or were totally abandoned). And while the number of surviving firms was unpredictable, it was always less than five. Of course, we have not modeled the possibility of re-entry or used more sophisticated notions of minimum efficient scale and do not claim that this approach would predict the number of survivors in any industry. Our main point is that the kind of simple, myopic firm behavior we have modeled can achieve equilibrium by changing the number of competitors.

2.4. Summary, Five is a Crowd

For the base case MSA model, five myopic profit-maximizing firms create a crowd. The same decision rules that lead fewer than five competitors to equilibrium produce turbulence or volatility in marketing budgets for five or more competitors. For firms with differential marketing effectiveness who exit when competitor spending leaves them with no profitable budget, our simulated industries shake out to fewer than five firms. Once again, five is a crowd.

It is almost impossible to consider these results without reflecting on whether they might be artifacts of a highly abstract model of competition, or, whether such turbulence has parallels in the world of competition between real firms. One “prediction” of the model is that markets with more than four competitors will exhibit volatility akin to the turbulence associated with “shakeouts.”
The actual phenomenon of shakeouts does not appear to have an extensive amount of empirical research, but Lambkin (1976) argued that a period of industry turbulence preceded shakeouts. Henderson (1976) claimed that “a stable competitive market never has more than three significant competitors, the largest of which has no more than four times the share of the smallest.” More recently, this observation was echoed by Sheth and Sisodia, (2002) who have written:

“With startling regularity, we have found that the number of dominant players in each industry is confined to three. Any other number, greater or smaller, is usually a temporary aberration.”

The “rule of three” is a hypothesis or what some might call a “stylized fact.” Any attempt at empirical validation will benefit from analyses that help us select the most appropriate industries and time periods that are likely to exhibit the turbulent budgeting behaviors in our models. Of course, we do not suggest that our model proves that spending coordination problems alone will cause shakeouts, only that they might be a contributing factor. The other side of this coin is to ask what kinds of changes are needed to prevent/eliminate this dynamic instability of market spending with myopic, profit-maximizing firms when shares are determined by the basic MSA model. It has been suggested to us that the same kind of myopic profit maximizing behavior might lead to inability for other (non-MSA) models of buyer behavior. We have no reason to believe the MSA model is unique in this respect. On the other hand, we have been unable to identify other simple models that would constrain total market shares to 1.0 and allow firms to identify an optimal marketing budget.

3. Partial Adjustment to Optimal Levels

Schmalensee (1976) observed that “as is usually the case in difference equation analysis, stability cannot be guaranteed for arbitrary positive speeds of adjustment.” But whereas Schmalensee (1976) explored conditions for “the possibility of local stability” defined as “the existence of a set of positive [adjustment speeds] such the equilibrium is locally stable,” we turn the question around and look for the combinations of adjustment speeds and number of firms that produce turbulence.

Following Schmalensee, we use k, a partial adjustment parameter, that regulates the degree to which sellers change marketing expenditures period-to-period. Specifically, if \((X' - X)\) is the difference in the profit maximizing budget at \(t\) and actual spending at \(t\), with partial adjustment the firm will change the budget by \(k(X' - X)\) in period \(t+1\). This finding is consistent with work by Kopel (1997) and Curie and Metcalfe (2001), showing that the path (and sometimes level) of a decision variable may depend on the adjustment speed of the market.

There are a number of ways \(k\) might be interpreted. For example, \(k\) might represent the commitment of firms to a particular market. Fully committed firms will adjust all the way to the optimal budget \((k = 1)\), and less committed sellers will hedge their bets \((k < 1)\). Parameter \(k\) might also be interpreted as a measure of the ease with which budgets can be ramped up or down in a single period. In what follows we will refer to \(k\) as the speed of adjustment.

By dampening the speed with which firms adjust spending to myopic optimal levels we can increase the number of firms that are able to converge to Nash Equilibrium. If \(k\) is small enough, any number of sellers can converge. Specifically, our simulations suggest that convergence to the NE will occur whenever \(km \leq 4\). This observation helps explain the five-is-a-crowd phenomenon. Too many sellers adjusting spending too quickly is what gets in the way of convergence. To observe convergence we can either start with less than five sellers, shake down to less than five sellers, or look for sellers who partially adjust their spending.

With partial adjustment, we also have the opportunity to explore what happens between four and five. Figure 3 shows two new spending patterns we found, a two-period cycle (at \(k = 0.9\) and \(m = 5\)) and a four-period cycle (at \(k = 0.47\) and \(m = 10\)). In the two-period cycle, sellers bounce back and fourth between two spending values and never find the NE. In the four-period cycle,
the sellers cycle repeatedly through the same four spending values. Notice that in both cases $km$ is greater than four and less than five. So for $km$ greater than four, we do not see convergence. Our observation that five is a crowd might now be refined to “more than four” is a crowd.

4. Analytical Explanation of Five is a Crowd

4.1. Different Starting Values

It is clear that starting $m$ symmetric firms at the same budgets will mean that all future spending will be synchronized. What is perhaps not so intuitive is that starting $m$ symmetric firms at different budgets will also lead to synchronized spending in our simulations. The synchronization occurs regardless of whether the firms converge to equilibrium, oscillate around the equilibrium, or follow some other pattern. Even five firms converge surprisingly quickly to a common spending value, which then follows the chaotic pattern illustrated in Figure 1D. (After only five periods, the range in the spending of five firms dropped to about 1 percent of total industry spending regardless of the range in initial spending.) So, whereas total spending is chaotic for a five-firm industry, there is strict order within that chaos. Firms quickly begin to move in lock-step – even if those steps are chaotic. Synchronization of spending (parallel peaks and valleys) also occurred in our simulations of sellers with differential marketing effectiveness.

Synchronization is not inevitable for all models of seller behavior in an MSA market. Kopel et al. (2000), for example, showed that synchronization does not occur when two symmetric firms update their spending based on profit achieved in the previous period and the elasticity of attraction to spending is small. Although our simulations convinced us that synchronization occurs in our base case model, we have not conducted the more rigorous analysis of this question as described in Bischi and Gardini (2000). An advantage of assuming synchronization is that it allows us to explore the reasons for the dynamic instability observed in our simple model of MSA-based competition.

4.2. The MSA Model, Competition, and the Logistics Equation

Given that firms quickly synchronize spending, we can learn about the entire dynamic system by studying a representative firm. Rather than simulate the spending of $m$ firms, we can explore a simple recursive equation that describes the spending of a representative firm. For $m$ synchronized firms using partial adjustment $k$, the recursive equation (or map) for firm spending is as follows [2]:

$$X_{t+1} = \frac{[k^2 M C (m - 1) X_t]^{1/2} -(km - 1)X_t}{(km - 1)^2}.$$  \hspace{1cm} (11)

A useful technique in the analysis of dynamic systems is to create a z-standardized metric, $Z_t$, for the decision variable. The transformation provided by equation (12) sets the minimum value of the decision variable to zero and the maximum to 1:

$$Z_t = \frac{X_t^{k^2 (m - 1) MC Z_t^2}}{(km - 1)^2}.$$  \hspace{1cm} (12)

Substituting into (11), we obtain

$$Z_{t+1} = [\mu Z_t (1 - Z_t)]^{1/2},$$  \hspace{1cm} (13)

where $\mu = km - 1$. This new equation is “standardized” in the sense that when $X_t = 0, Z_t = 0$ and when $X_t$ equals its maximum feasible value of $k^2 (m - 1)(MC)/(km - 1)^2, Z_t = 1$. (The maximum feasible value of $X_t$ is the value that insures $X_{t+1} \leq 0$.) We will refer to map (13) as the Standard Competitive Spending (SCS) map.

The nonlinear transformation from $X$ to $Z$ simplifies subsequent analysis because the dynamical properties of the new SCS map are often summarized in a bifurcation diagram. The bifurcation diagram plots the set of possible values the system can obtain as a function of the single parameter $\mu$. The maps (11) and (12) are said to be conjugate by (13) (see Martinelli 1999, p. 50).

The properties of a nonlinear dynamic system such as the SCS map are often summarized in a bifurcation diagram. The bifurcation diagram plots the set of possible values the system can obtain as a function of $\mu$, the system parameter. We present the bifurcation diagram of the SCS map in Figure 4. Also, we match the five spending graphs from the previous sections to the appropriate point in the bifurcation diagram. These spending graphs correspond to five vertical slices of the bifurcation diagram. Note that the spending graphs and the bifurcation diagrams use different scales for the vertical axes.

From the bifurcation diagram, we see that the system converges to a single value whenever $\mu < 3$. This represents the convergence of seller spending to the NE whenever $km < 4$. At $\mu = 3$ (km = 4), the system undergoes its first bifurcation. For $\mu > 3$ but less than 3.650, the system will oscillate between two values. The transition between convergence and two-period oscillation occurs exactly at $\mu = 3$, the first bifurcation. For the base case model (with $k = 1$) this bifurcation corresponds to $m = 4$. The “slow convergence” for the 4-firm industry is now seen as a transition point between convergence and 2-period oscillation.

At $\mu = 3.650$ (approximately) the system bifurcates once again. Just to the left of 3.650 we observe 2-period oscillation and just to the right we observe 4-period oscillation. The spending graph for $m = 10$ and $k = 0.47$ (corresponding to $\mu = 3.7$) illustrates the 4-period oscillation. At approximately 3.753 the system bifurcates once more and enters a small region of 8-period oscillation.

The graph ends at $\mu = 4$, the point where the SCS map becomes infeasible. At $\mu = 4$ the system takes on values bounded by 0 and 1 as illustrated in the spending graph for $k = 1$ and $m = 5$. Although the system is chaotic, it stays between 0 and 1. For $\mu > 4$, $Z$-values will move outside of the (0,1) range and spending will become negative and infeasible.
Readers familiar with nonlinear dynamical systems may notice the similarity between the dynamic properties of the SCS map and the well known standard logistic (SL) map:

\[ Y_{t+1} = \mu Y_t (1 - Y_t). \]  

(14)

The SCS map (13) is the same as the SL map (14) with a square root added to the right hand side.

Both the SCS map and the SL map belong to a class of unimodal and concave maps on the unit interval. Moreover, like the SL map, the SCS map is symmetric with
respect to the symmetry axis $Y = 1/2$. This gives rise to many similarities in the dynamic behavior, which is apparent by comparing the bifurcation diagrams in Figure 5. Note that the $Y = 1/2$ axis of symmetry applies to maps (13) and (14) - not to these bifurcation diagrams.

As the diagrams show, both equations bifurcate first at $\mu = 3$ and become infeasible at $\mu > 4$. The second and third bifurcations are different, however. In spite of this difference, the behavior of the two equations appears qualitatively very similar, as shown in Figure 5, the comparison of the bifurcation diagrams.

4.3. The Conditions for a Crowd

If by “crowd” we mean an industry that fails to converge to the NE, $\mu > 3$ in the SCS map means we have a crowd. Because this corresponds to $km > 4$, we might also express this as $m > 4/k$ is a crowd.

In the language of dynamical systems, our “crowd” conditions correspond to the first bifurcation. For the SCS map, the first bifurcation is the point where the fixed point $[3] \mu/(1 + \mu)$ transitions from an attractor to a repeller. Whether a fixed point is an attractor or repeller depends on the first derivative of the map evaluated at the fixed point. If the absolute value of the derivative is less than 1, the fixed point is an attractor. If it is greater than 1, the fixed point is a repeller. If it is equal to 1, the fixed point is called neutral or indifferent (see Devaney 1992, p. 36).

For the SCS map, the first derivative evaluated at the fixed point is $(1 - \mu)/2$. The absolute value of this derivative is less than unity for $\mu < 3$, is equal to unity for $\mu = 3$, and is greater than unity for $\mu > 3$. In subsequent sensitivity analyses, we will use this analytical method to find the first bifurcation – the conditions leading to a crowd – in more complicated systems.

5. Sensitivity of the MSA’s Dynamical Properties to Selected Market Characteristics

In this section we present the results of eight modifications to our base case model and evaluate the effect of these changes on the particular dynamic properties that are our focus: the value of $m$ that represents the threshold of turbulence properties. The eight model characteristics are:

1. elasticity of market attraction with respect to marketing spending,
2. multiple marketing mix elements,
3. share carryover effects,
4. exogenous growth (not affected by marketing spending),
5. endogenous growth (market size affected by industry marketing spending),
6. the introduction of “noise” into the budgeting process,
7. the replacement of simultaneous with sequential decisions, and
8. introduction of firms “learning” to anticipate competitive spending.
Kopel et al. (2000) have pointed out that initial spending can affect the dynamic properties of firms spending in an MSA market. Their firms, however, were not myopic optimizers. Using simulation to explore varying initial spending amounts for the base case, we always observed a synchronization emerge. Our simulations of the threshold model used identical initial spending amounts and we again always observed that a synchronization emerged.

For the sensitivity analyses that follow, we will assume identical initial spending amounts. While our intuition is that varying initial spending amounts will not affect the dynamical properties of the system, additional work will be required to verify this conjecture.

5.1. Elasticity of Attractiveness to Marketing Spending

The base case assumes attraction is a linear function of spending \((A_i = X_i)\). Frequently, researchers use a function in which attraction is proportional to spending raised to a power \((A_i = X_i^\beta)\). This functional form is quite common and is used in what Cooper and Nakanishi (1988) call multiplicative competitive interaction (MCI) models. Notice that our base case MSA model is equivalent to the MCI model with one spending element with \(\beta = 1\). The parameter \(\beta\) is the elasticity of attraction with respect to spending – not to be confused with the elasticity of share with respect to spending. For \(\beta < 1\), this function will further depress returns to marketing spending, beyond the diminishing returns inherent in the basic attraction formulation.

The point elasticity of share with respect to increases/ decreases in spending equals \(\beta \times (1 - \text{current share})\) [4]. Thus even for \(\beta = 1\), share elasticities are less than 1. For example, if \(\beta = 0.5\) and current share were 0.4, point elasticity of share with respect to marketing spending would equal 0.5 \(\times (1 - 0.4)\), or 0.3.

If \(A_i = X_i^\beta\), we would like to know the conditions under which the system will converge to the NE. Equivalently, we want to find the conditions under which the fixed point of the new dynamic system is an attractor (repeller).

To derive the relationship between \(X_{i,t+1}\) and \(X_i\) for \(m\) synchronized firms, we begin with the equation for net profit as a function of the spending of firm \(i\) given the spending \(X_{i,t}\) of each of \(m\) other firms:

\[
NP = MC \times X_i^\beta - \sum_{j=1}^{m} X_j^\beta - X_i^\beta.
\]

Taking the derivative of \(NP\) with respect to \(X_{i,t}\), setting to zero and simplifying gives the following optimality condition:

\[
MC(\beta)(X_{i,t}^{\beta-1}(m - 1)X_i^{\beta} = [(X_{i,t}^{\beta}) + (m - 1)X_i^{\beta}]^2. \quad (16)
\]

Although we cannot solve this equation for \(X_{i,t}\) in terms of \(X_i\), we can differentiate both sides and rearrange to obtain

\[
\frac{\partial X_{i,t}^\beta}{\partial X_i} = \frac{2((X_i^{\beta}) + (m - 1)X_i^{\beta})}{(m - 1)MC(X_i^{\beta-1} - 1)(X_i^{\beta-1} - 2X_i^{\beta}) + (m - 1)MC(X_i^{\beta-1})}. \quad (17)
\]

We are only interested in this derivative evaluated at the NE where \(X_{i,t}^* = X_i = MC(\beta)(m - 1)/m^2\). Substituting the NE for both \(X_{i,t}\) and \(X_i\) and simplifying yields

\[
\frac{\partial X_{i,t}^\beta}{\partial X_i} \bigg|_{NE} = \beta(\beta(m - 2))/m(\beta - 1) - 2\beta. \quad (18)
\]

With partial adjustment for synchronized firms

\[
X_{i,t+1} = kX_i^* + (1 - k)X_t
\]

and the derivative of \(X_{i,t+1}\) with respect to \(X_i\) evaluated at the NE becomes

\[
\frac{\partial X_{i,t+1}^\beta}{\partial X_i} \bigg|_{NE} = k\beta(\beta(m - 2))/m(\beta - 1) - 2\beta + (1 - k). \quad (20)
\]

The fixed point is an attractor (the NE is locally stable) if this derivative is between -1 and +1. If \(k + 2\beta \leq 2\), the fixed point is an attractor for all \(m\). If not, the fixed point is an attractor if \(m < (4\beta)/(k + 2\beta - 2)\).

In summary, there can not be a crowd if \(k + 2\beta \leq 2\). If \(k + 2\beta > 2\), \(m\) greater than \((4\beta)/(k + 2\beta - 2)\) becomes a crowd. Notice that at \(k = 1\) and \(\beta = 1\), \(m\) greater than 4 is a crowd. For \(k = 1\) and \(\beta = 0.6\), \(m\) greater than 12 is a crowd.

It is tempting to look for published research applying the MSA model and compare the values of \(\beta\) estimated (often around 0.3) to the above limits. However, we do not believe this direct comparison with published studies is possible. The reason is that published studies generally focus on a single marketing mix element (such as advertising) and/or model multiple mix elements, each with a different elasticity. Such studies will have much lower elasticities than the single category for marketing spending that we have used. Our next section demonstrates why this is true.

5.2. Multiple Spending Elements

Suppose the firm’s total spending is divided across \(n\) distinct elements (for example: sales force, advertising, promotion, distribution, etc.). Suppose further that attraction is a multiplicative function of spending on each element with constant elasticities:

\[
A = X_1^\beta X_2^\beta ... X_n^\beta.
\]

It can be shown that the profit-maximizing firm will spend equally across all spending elements. Consequently total attraction will be \((X/n)^\beta\) where \(X\) represents total spending. Net profit will be given as

\[
NP = MC \times \frac{X_1^\beta \sum_{i=1}^{n} X_i - X_i}{X_1^\beta + \sum_{i=1}^{n} X_i}.
\]

The resulting dynamic system will be equivalent to one with a single spending element with elasticity \(n\beta\). So, for
example, if attraction elasticities in a four-component model are typically 0.2, the equivalent attraction elasticity for total spending is 0.8. This 0.8 attraction elasticity would correspond to a share elasticity of 0.48 for a firm with a share of 0.4 [0.8×(1 – 0.4)] [5].

5.3. Share Carryover Effects

Suppose \( \lambda \) of this period’s share carries over to next period for all firms in the market:

\[
S_{i,t} = \lambda S_{i,t-1} + (1 - \lambda)[A_{i,t}/(A_{i,t} + \sum A_{i,t})],
\]

(23)

where \( \sum A_{i,t} \) represents the sum of the attractions of firms other than \( i \).

One way to think of this is that each period only \( (1 - \lambda) \) of the buyers are “in the market” and influenced by spending. As a consequence, carryover effects dampen the amount by which market shares change period-to-period. Carryover effects slow down the buyers “speed of adjustment” to changes in spending. This makes the model more applicable to later stages of market evolution when customers may have developed loyalties and the competition among sellers is for a smaller portion of the overall market.

Equation (23) describes the specific “glide path” shares follow as they react to changes in spending. A continuous-time model for share glide paths is the Lanchester map, the dynamical properties of the system do not change. As reflected in the transformation to the SCS map, the dynamical properties of the system are not a function of MC (the size of the market). Changing MC over time in our simulations did not affect the threshold of turbulence as long as the m firms set spending using a common anticipated market size.

5.4. Exogenous Market Growth

Since we have hypothesized that the conditions modeled are most likely to occur at the early stages of the product life cycle, this begs the question of how market growth might affect the m-threshold of dynamic stability. First we examine exogenously determined growth (such as that enabled by new technologies).

If sellers anticipate market growth and plan period \( t+1 \) spending based on a common forecast of market size, the dynamical properties of the system do not change. As reflected in the transformation to the SCS map, the dynamical properties of the system are not a function of MC (the size of the market). Changing MC over time in our simulations did not affect the threshold of turbulence as long as the m firms set spending using a common anticipated market size.

5.5. Endogenous Market Growth

Suppose the total market contribution is proportional to industry spending raised to the power \( \theta \), then

\[
MC_t = B\left(\sum_i X_{i,t}\right)^\theta.
\]

(27)

Here \( B \) is a constant. Our base case had \( B = MC \) and \( \theta = 0 \).

To derive the relationship between \( X_{t+1} \) and \( X_t \) for m synchronized firms, we begin with the equation for net profit as a function of the spending of firm \( i \) given the spending of each of \( m-1 \) competing firms:

\[
NP(X_{i,t}) = B[X_{i,t} + (m - 1)X_{i,t}^\theta] \cdot [X_{i,t} + (m - 1)X_{i,t}]^{-1} - X_{i,t}.
\]

(28)

Taking the derivative of \( NP \) with respect to \( X_{i,t} \), setting to zero and simplifying gives the following optimality condition:

\[
B[X_{i,t} + (m - 1)X_{i,t}] = (m - 1)X_{i,t}^\theta .
\]

(29)

Although we can not solve this equation for \( X_{i,t} \), in terms of \( X_{i,t} \), we can differentiate both sides and rearrange to obtain

\[
\frac{\partial X_{i,t}}{\partial X_{i,t}} = \frac{B(m - 1) - (2 - \theta)(m - 1)X_{i,t} + (m - 1)X_{i,t}]^{1-\theta}}{(2 - \theta)X_{i,t} + (m - 1)X_{i,t}^\theta} .
\]

(30)

We are only interested in this derivative evaluated at the NE where \( X_{i,t}^{NE} = X_{i,t} = B[\theta + m - 1]/(\theta + m - 1) \). Substituting the NE for both \( X_{i,t} \) and \( X_{i,t} \) and simplifying yields

\[
\frac{\partial X_{i,t}}{\partial X_{i,t}} = \frac{(m - 1)(m - 2\theta - \theta + m - 1)}{(2 - \theta)(\theta + m - 1) - \theta(m)} .
\]

(31)

With partial adjustment for synchronized firms

\[
\frac{\partial X_{i,t}}{\partial X_{i,t}} = \frac{(m - 1)(m - 2\theta - \theta + m - 1)}{(2 - \theta)(\theta + m - 1) - \theta(m)} + (1 - k) .
\]

(32)
The fixed point is an attractor (the NE is locally stable) if this derivative is between -1 and +1. Some algebraic manipulation reveals that the fixed point is an attractor if

\[ km < \frac{2(1 - \theta)(\theta + m - 1) - 2\theta m}{(1 - \theta)(\theta + m - 1)}, \]

assuming the numerator is positive (which is true for \( \theta < 1 \)).

This equation summarizes the crowd conditions for sellers using partial adjustment in an elastic total market. When \( \theta = 0 \), the above condition simplifies to \( km < 4 \) as shown earlier. When \( \theta > 0 \), \( km = 4 \) becomes a crowd. When the sellers compete not only for share but also to grow the total market, spending competition is more intense and spending adjusts a little faster. Whereas growth increases adjustment speed and make convergence a little more difficult and crowding a little more likely.

5.6. Error in the (Optimal) Budgeting Process

Young (1998) observed that a “remarkable feature of stochastic dynamical systems is that their long-term (asymptotic) behavior can differ radically from the corresponding deterministic process no matter how small the noise term is. The presence of noise, however minute, can qualitatively alter the behavior of the dynamics. But there is also an unexpected dividend. Since such processes are often ergodic, their long-run average behavior can be predicted much more sharply than that of the corresponding deterministic dynamics, whose motion usually depends on the initial state.”

Does including error in the firms’ spending affect our results? Certainly real firms do not have perfect knowledge of how buyers will respond to marketing nor do they have perfect control over how much they actually spend in any period.

To investigate this possibility, we extended the model to allow actual spending for each firm in each period to be equal to optimal spending times a multiplicative error:

\[ X_t = X_t^* e^\epsilon \]  

(34)

where \( \epsilon \) follows a normal distribution with mean 0 and variance \( \sigma^2 \).

In our base case, introducing error into the budgeting decision led to instability for \( m > 4 \) as well as interrupted the smooth movement to equilibrium for \( m = 4 \) and made the overall patterns less recognizable between simulation runs of the same parameters. This particular kind of error did not achieve the “stochastic stability” envisioned by Young (1998).

5.7. Sequential Decisions

The \( m \) firms in the model decide and implement spending simultaneously. It is this simultaneity of decisions that contributes to the coordination challenge and resulting spending turbulence. If firms decide sequentially rather than simultaneously, simulated industries with 30 firms converged. Sequential decisions represent the opposite extreme from simultaneous decisions. With sequential decisions, each firm adjusts every \( m \) periods and only one firm adjusts in each period. Industries in which firms “take turns” adjusting do not face the coordination challenge.

An interesting area for research is whether and why firms tend to make decisions at the same time. There are several aspects of markets that might cause simultaneity to emerge, even if firms started on different decision schedules. Regulatory reporting requirements, seasonal influences on sales, retailer planning programs, syndicated market research are just some of the influences that might favor simultaneity. With 2 or 3 firms it seems plausible that firms might decide sequentially. Two firms might easily take turns adjusting their marketing budgets, for example. As the number of firms increases, however, strict sequential decisions become less plausible and simultaneous decisions more plausible.

5.8. Anticipation of Competitive Spending

In the base case, each firm selects its budget for period \( t+1 \) based on market conditions in period \( t \) (which are a function of competitors’ spending in period \( t \)) but without knowledge of (or explicit anticipation of) competitors’ spending in period \( t+1 \). What changes if we assume that firms observe competitor spending and learn to predict future spending on the basis of historical patterns? We tested two forms of “learning” to anticipate competitive spending. Rules that lower the speed of spending adjustment raise the \( m \)-threshold and those that increase the speed of adjustment have the opposite effect.

For example, if each firm uses a two-period moving average to anticipate period \( t+1 \) competitive spending, the speed of spending adjustment is lower. As a consequence, 7 simulated firms converge, 9 firms do not, and 8 firms converge for some initial conditions and not for others.

On the other hand, if firms anticipate next period’s competitive spending as \( X_t^* + (X_{t-1} - X_{t-2}) \), then the \( m \)-threshold is lower (than four). This simple linear extrapolation of competitive spending prevented three simulated firms from converging (but allowed two to converge).

5.9. Attraction as a Function of Price

One method for incorporating price in the MSA model is to use an attraction that is proportional to \( (V-P)^\alpha \) (see Gruca/Kumar/Sudharshan 1992). Here \( V \) represents the buyers’ reservation price (the maximum amount each buyer is willing to pay), \( P \), is the price selected by firm \( i \), and \( \alpha \) is a constant elasticity assumed to lie between zero and one.

Assume further that rather than competing for a fixed amount of dollar contribution \( M \), firms compete for a
fixed number of units represented by SP, the buyers’ sales potential. To isolate the dynamics of MSA pricing competition, we examine the convergence conditions for a market in which there is only pricing competition and no spending competition.

To derive the relationship between $P_{i,t}$ and $P_j$ for $m$ synchronized firms, we begin with the equation for net profit as a function of the price of firm $i$ given the prices of each of $m$-1 competing firms:

$$NP(P_{i,t}) = SP(P_{i,t} - C)(V - P_{i,t})^\alpha [(V - P_{i,t})^\alpha + (m - 1)(V - P_{i,t})^\alpha]^{1/\alpha}.$$  

(35)

Here $C$ is the variable cost per unit. Taking the derivative of $NP$ with respect to $P_{i,t}$, setting to zero and simplifying gives the following optimality condition:

$$(V - P_{i,t})^{1+\alpha} + (m - 1)(V - P_{i,t})^{2\alpha} [(V - P_{i,t}) - \alpha (P_{i,t} - C)] = 0.$$  

(36)

Although we can not solve this equation for $P_{i,t}$ in terms of $P_{j,t}$, we can differentiate both sides and rearrange to obtain

$$\frac{\partial P_{i,t}}{\partial P_{j,t}} = \frac{(V - P_{i,t})^{1+\alpha} - \alpha (m - 1)(V - P_{i,t})^\alpha (P_{i,t} - C)}{(1 + \alpha)((V - P_{i,t})^{2\alpha} + (m - 1)(V - P_{i,t})^{2\alpha})}.$$  

(37)

We are only interested in this derivative evaluated at the NE where $P_{i,t} = P_{j,t} = P_{NE}$,

$$p_{NE} = \frac{mV + \alpha(m - 1)C}{m + \alpha(m - 1)}.$$  

(38)

Substituting the NE for both $P_{i,t}$ and $P_{j,t}$ and simplifying yields

$$\left.\frac{\partial P_{i,t}}{\partial P_{i,t}}\right|_{NE} = \frac{-\alpha}{(1 + \alpha)m}.$$  

(39)

The fixed point is an attractor (the NE is locally stable) if this derivative is between -1 and +1. Since $\alpha$ is between zero and one, this derivative will always be greater than -1 and less than 1. Consequently, the NE is always locally stable and the fixed point is always an attractor.

Notice that in contrast to spending competition, this derivative gets flatter (closer to zero) as $m$ increases. For MSA pricing competition, more firms mean quicker convergence – the opposite effect of more competitors for non-price competition [6].

By way of explaining these results, we showed that the recursive equation describing firms’ spending behavior is very similar to the standard logistic map – an equation used to describe the booms and busts of biological populations. We were surprised that a simple model of competition in an MSA market yields an equation so similar to the standard logistic map. In hindsight we find many connections that were not apparent when we began this analysis. It is no longer so surprising that the behavior of firms competing for share of market is similar to that of living populations competing for survival in a world with limited carrying capacity.

According to May (1976) the standard logistic map was first used in 1845 by Verhulst to model the development of populations in environments with limited resources. Just as organisms multiply quickly when there are very few of them and die off when their numbers approach the carrying capacity of their environment, so too does marketing spending expand when not many dollars are spent and contract when industry spending approaches $MC$, the limit the industry can support. The $\mu$ parameter in the standard logistic map represents the unencumbered “growth” rate of the biological population. In our model, the “growth” rate for industry spending depends on the number of competitors ($m$) and their speed of adjustment ($k$). The more firms and the faster they adjust, the faster industry spending “grows” in times with low spending and contracts in times with high spending.

### 6.1. $m$-Threshold of Turbulence

The main focus of our analysis has been on understanding where and why there is a threshold in the number of competitors that create turbulent spending patterns. The speed of industry spending adjustment helps explain the phenomenon. The $m$-threshold depends on the “aggressiveness” of the industry. The faster the firms adjust spending, the lower the threshold number of competitors that leads to turbulence. In the base case, adjustment speed (of the industry) depends only on $m$. The partial adjustment parameter, $k$, lowers the speed of adjustment directly and lowers the $m$-threshold of turbulence. The sensitivity results in Section 5 explored several additional market characteristics for their influence on the $m$-threshold and these results are summarized in Table 2.

Elasticities of attraction with respect to total spending less than 1.0 were shown to raise the $m$-threshold. A comparison of single, versus multiple marketing mix elements demonstrated that the relevant elasticity for comparison is the sum of the marketing mix elasticities for the specific situation where elasticities are equal. Carry-over effects, as modeled, did not affect the $m$-threshold of turbulence or the type of patterns observed, only the range of spending (lower for higher carryover effects). For exogenous growth, again, no change in the $m$-threshold was observed. Endogenous growth (driven by aggregate marketing spending) did change the result, working to lower the $m$-threshold. Introducing error into the myo-
pic-optimal spending decision only changed the basic patterns for \( m = 4 \). The error worked to prevent four firms from converging and increased turbulence. If decisions are not simultaneous the \( m \)-threshold is higher.

Simultaneity of decisions, an assumption of convenience in our models, has been referred to as “synchrony.” Synchrony has been shown to emerge in a variety of real-world settings [7]. It is clearly a necessary condition for the “five-is-a-crowd” phenomenon. We believe the plausibility of this assumption in business competition is an interesting area for further inquiry.

6.2. Implications for Researchers and Managers

We believe that the complicated dynamical properties illustrated in this paper should be neither easily dismissed mathematical curiosities nor regarded as model defects to be fixed. Just as researchers in the life sciences have come to accept these properties as important descriptors of the actual behaviors of living populations, so too should we consider reaching the same conclusion about markets. This work suggests that turbulence in real markets may be caused not only by exogenous random factors, but also by the simple behavior of firms acting in a nonlinear system.

A potentially important implication for managers is that failing to project competitive actions and attending exclusively to customer response, in early stages of the life cycle might lead to turbulent spending when the number of competitors exceeds a critical threshold value. This is somewhat at odds with the more prevalent notion that competitive “wars” are typically caused by paying too much attention to competitors and not enough to customers. This reinforces the idea that an important managerial function is attending to competition and anticipating their actions and reactions.

Our simulations include firms that use relatively simple optimization rules that result in collective behavior that surprised us with its complexity and unpredictability. Young (1998) writes “one of the central messages of the pure theory of exchange, for example, is the ability of prices and markets to coordinate economic activity without assuming that the agents are anything more than naïve optimizers acting on limited information.” Naïve optimizers function “just fine” in a competitive environment of four or fewer firms. Once we cross the \( m \)-threshold, we will need more than naïve optimizers to build a stable simulated industry. Either some simulated firms will leave or the remaining firms must learn to be something other than naïve optimizers.

### 6.3. Empirical Verification of the Model Predictions

Our model assumes that firms use customer knowledge to select their optimal spending budget – but do so without regard for future competitive movements. Is this a realistic assumption? Certainly real firms often use test markets to estimate customer response as a precursor to selecting spending for the next budgetary cycle. In this process, what assumptions do firms make about the competition? Urbany et al. (2000) suggest that most firms fail to make any explicit assessment of competitive reactions. "Consideration of potential competitive reactions ... still has an incidence below 20 percent." Urbany et al. report a survey with the following frequency of mention of decision factors: 60 % of the time customer demand is considered, 54 % of the time past or current competitive behavior is considered, 21 % of the time expected future competitive behavior in taken into consideration, and 0 % mentioned expected future competitor reactions as decision factors. Our base case model corresponds to the 60 % and 54 % behaviors.

Even if there is evidence that managers often fail to consider potential competitive reactions, there is still a question of whether actual markets exhibit the kind of instability exhibited in our simulated industries. We offer no direct evidence on this question and believe that collecting such evidence will be a challenge. However, our belief is that markets and marketers in the real world do often over-react to threats and opportunities. Airline price wars, dot.com advertising budgets, and hotel/office space capacity booms and busts are examples. Often these are attributed to lags and feedback effects. Possibly, some of these are due to inherently unstable numbers of competitors targeting the same market. Many firms who were convinced of the need to move at "Internet speed" are now licking their wounds and developing respect for what they don’t know about their competition.

In spite of the difficulty of testing the model, we believe this is a very simple model that may throw some light on the kind of behavior that surprised us with its complexity and unpredictability. Young (1998) writes “one of the central messages of the pure theory of exchange, for example, is the ability of prices and markets to coordinate economic activity without assuming that the agents are anything more than naïve optimizers acting on limited information.” Naïve optimizers function “just fine” in a competitive environment of four or fewer firms. Once we cross the \( m \)-threshold, we will need more than naïve optimizers to build a stable simulated industry. Either some simulated firms will leave or the remaining firms must learn to be something other than naïve optimizers.
"The piling on is typical of American marketing, particularly in fields with little or no sales growth ... as the new ... When Five is a Crowd in the Market Share Attraction Model

M A R K E T I N G·J R M·1 / 2 0 0 5

Henderson’s

Will only three survive in this category? Even before significant competitors"

stable competitive market never has more than three sig-

Notes

[1] As we see it, this exit assumption is consistent with assump-
tion 5. Our sellers are myopic profit maximizers. If our sellers see no way to make a profit, the myopic action would be to abandon the industry. Clearly real sellers are never this myopic and must contend with fixed costs of exit and reentry.

[2] Since we are modeling symmetric and synchronized firms, we require many years to reach equilibrium unless the exit requires many years to reach equilibrium unless the leader chooses to hold his share during the high growth phase of the product life. However, the rule appears to be inexorable." (Henderson 1976).

The analyses in this paper offer one possible explanation for the number of industry competitors being limited to three or four - in any event, less than five.

[6] Since price directly affects MC, pricing competition would affect the NE level of non-price spending. Would industries characterized by more intense price competition accommodate a larger number of competitors than industries in which non-price competition is more important? Or, might the result depend more on the sequence with which competitors resorted to price and non-price actions?

[7] “The study of synchrony, sync provides a crucial first step what’s coming next in the study of complex nonlinear systems, where the oscillators are eventually going to be replaced by genes and cells, companies and people.” (Strogatz, 2003, p. 247).

References


Farris/Pfeifer/Reibstein/Nierop, When Five is a Crowd in the Market Share Attraction Model


... certainly a must for anyone who does a lot of business translation